Structural Completion of Pattern Field Theory's and Turing's Morphogenesis Under Admissibility and Logarithmic Lift

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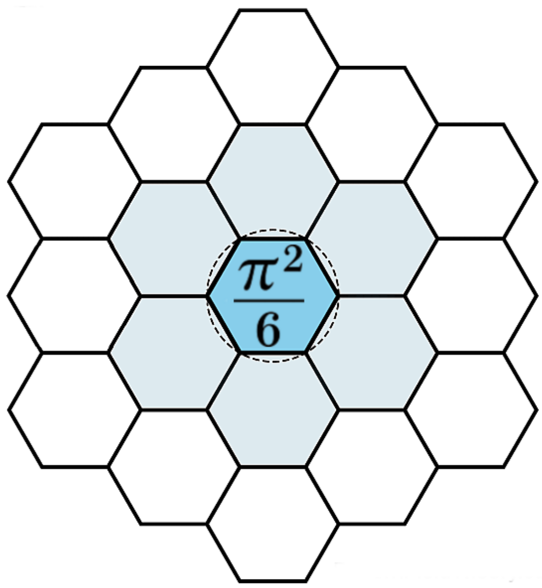

Abstract

Pattern Field Theory (PFT) defines morphogenesis as a transition from continuous morphogenic fields to discrete persistent structure governed by admissibility, basin geometry, depth stratification, and layered locking on the Allen Orbital Lattice. This paper formalizes that morphogenic transition system as a computable operator algebra. Continuous morphogenic fields are treated as coordinate-route routing layers, while persistent structures are treated as identity-route structure states. We define and integrate the Admissibility operator, the EQUI equilibrium operator, the Phase Alignment Lock (PAL) temporal coherence constraint, the Logarithmic Lift depth operator, and the Locking commit operator. A theorem chain establishes that PFT morphogenesis converges to discrete basin-locked identity structures. In two-dimensional isotropic basins, the stable identity-route structure attractor class is proven to be hexagonal and identified with the QuantaHex regime on the Allen Orbital Lattice. As a corollary, Turing-type reaction-diffusion systems are shown to be a special case of coordinate-route morphogenesis that becomes structurally closed under the PFT operator set.

This work forms part of the broader Pattern Field Theory research program, which investigates structural organization across substrate, dynamical, geometric, and morphogenic levels.

The present paper focuses specifically on morphogenic structural closure under admissibility and logarithmic lift. It does not assume a particular gravitational derivation or emergent spacetime construction. It operates at the level of structural evolution and morphogenic closure, independent of any specific substrate dynamics or field-theoretic realization. Instead, it establishes operator-level closure principles for transitions from continuous routing layers to discrete persistent identity structures.

Scope

This paper formalizes morphogenesis as defined by Pattern Field Theory. The objective is to present a complete operator-level description of the transition from continuous morphogenic fields to discrete persistent structures. Pattern Field Theory morphogenesis is treated as primary and autonomous. Turing-type reaction-diffusion systems are treated as a special case of coordinate-route morphogenesis that is structurally incomplete without the PFT operator set.

The goal is not to reinterpret Pattern Field Theory in terms of Turing systems, but to show that Turing systems are contained within the PFT morphogenic framework.

Morphogenic Fields and Routing Layers

Let a morphogenic field be a state function

$$\Phi : \Omega \times \mathbb{R}^+ \to \mathbb{R}^n.$$

Its coordinate-route evolution is given by a local update operator Δ plus spatial coupling, written abstractly as

$$\Phi_{t+1} = \Phi_t + \Delta(\Phi_t).$$

Definition 1 (Routing Layer)**.** *A routing layer is a continuous morphogenic field whose evolution explores configuration space without structural commitment.*

Routing layers implement coordinate-route dynamics. They generate patterns but do not define identity or persistence.

Coordinate Route and Identity Route

Definition 2 (Coordinate Route)**.** *A coordinate-route system is a morphogenic process whose state is fully described by its position in configuration space and which remains subject to continuous modification by the routing operator.*

Definition 3 (Identity Route)**.** *An identity-route structure system is a morphogenic state that has been committed by locking and is no longer subject to routing-layer modification.*

Proposition 1. *Routing layers operate in coordinate-route mode. Locked structures exist in identity-route structure mode.*

Allen Orbital Lattice Coordinate Charts

Definition 4 (AOL Axial Coordinates)**.** *The Allen Orbital Lattice is represented in two dimensions by axial integer coordinates*

$$\chi = (q, r) \in \mathbb{Z}^2$$

with six nearest-neighbor steps

$$\mathcal{N}_6 = \{(1,0), (0,1), (-1,1), (-1,0), (0,-1), (1,-1)\}.$$

Definition 5 (AOL Cube Coordinates)**.** *Equivalently define cube coordinates*

$$\hat{\chi} = (x, y, z) \in \mathbb{Z}^3, \quad x + y + z = 0$$

with the bijection

$$x = q, \quad z = r, \quad y = -q - r.$$

Definition 6 (Euclidean Embedding)**.** *Let $\ell > 0$ be the AOL lattice spacing. Define an embedding $\iota : \mathbb{Z}^2 \to \mathbb{R}^2$ by*

$$\iota(q, r) = \ell \begin{bmatrix} q + \frac{1}{2} r \\ \frac{\sqrt{3}}{2} r \end{bmatrix}.$$

The induced Euclidean distance between two AOL sites χ_1, χ_2 is

$$d_E(\chi_1, \chi_2) = \|\iota(\chi_1) - \iota(\chi_2)\|_2.$$

Definition 7 (AOL Graph Distance)**.** *Define the AOL graph distance by cube coordinates*

$$d_{\mathrm{AOL}}(\hat{\chi}_1, \hat{\chi}_2) = \frac{1}{2} \left(|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| \right).$$

Proposition 2. *For nearest neighbors $\chi_2 - \chi_1 \in \mathcal{N}_6$, one has $d_{\mathrm{AOL}} = 1$ and $d_E = \ell$.*

Definition 8 (Allen Orbital Lattice)**.** *The Allen Orbital Lattice* AOL *is the discrete identity-route structure substrate on which locked structures are embedded.*

Admissibility and Basins

Definition 9 (Admissibility Operator)**.** *Let $\mathcal{A}$ be a predicate on field states. A state Φ is admissible if and only if $\mathcal{A}(\Phi) = \mathrm{true}$. Transitions to non-admissible states are forbidden.*

Definition 10 (Basin)**.** *A basin is a maximal connected region of state space closed under admissible transitions.*

Proposition 3. *Admissibility induces finite basins and structural ceilings.*

Proof. If continuation outside a region violates $\mathcal{A}$, then admissible transitions are confined to that region. Since resources, coherence, and curvature margins are bounded, the region is finite in effective state volume. □

EQUI Constraint

Definition 11 (EQUI Operator)**.** *Let Φ define an embedding of a candidate identity-route structure into the Allen Orbital Lattice with local metric g_{ij} and curvature tensor K_{ij}. Define:*

$$\mathcal{E}(\Phi) = \int_{\Omega} \rho(\Phi)\, dV, \qquad \kappa_{\max}(\Phi) = \sup_{\Omega} \|K_{ij}(\Phi)\|, \qquad \sigma_{\max}(\Phi) = \sup_{\Omega} \|\Sigma_{ij}(\Phi)\|$$

where ρ is stored energy density and Σ_{ij} is structural load stress.

Then the equilibrium admissibility operator is defined as:

$$\mathrm{EQUI}(\Phi) = \begin{cases} 1 & \text{if } \mathcal{E}(\Phi) \le E_{\max} \ \wedge \ \kappa_{\max}(\Phi) \le \kappa_{\text{crit}} \ \wedge \ \sigma_{\max}(\Phi) \le \sigma_{\text{crit}} \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 4. *If* $\mathrm{EQUI}(\Phi) = 0$*, then Φ cannot be locked into a stable identity-route structure.*

Proof. If any bound is violated, the resulting structure exceeds admissible energy, curvature, or stress margins and necessarily diverges from basin stability under continuation. Therefore locking produces an unstable or destructive configuration. □

Curvature Accumulation and Structural Budgets

Let an identity-route structure candidate structure be represented on the AOL by a discrete scalar potential $\psi : \mathbb{Z}^2 \to \mathbb{R}$ and a discrete displacement field $u : \mathbb{Z}^2 \to \mathbb{R}^2$ (optional). Define the discrete gradient along an oriented edge $(\chi, \chi + \delta)$ with $\delta \in \mathcal{N}_6$ by

$$\nabla_\delta \psi(\chi) = \psi(\chi + \delta) - \psi(\chi).$$

Definition 12 (Discrete Laplacian on AOL)**.** *Define the six-neighbor discrete Laplacian by*

$$(\Delta_{\text{AOL}}\psi)(\chi) = \sum_{\delta \in \mathcal{N}_6} \left(\psi(\chi + \delta) - \psi(\chi)\right).$$

Definition 13 (Curvature Proxy and Accumulation)**.** *Define a curvature proxy*

$$\kappa(\chi) = \frac{1}{\ell^2} (\Delta_{\text{AOL}}\psi)(\chi),$$

and define curvature accumulation energy

$$\mathcal{K}(\psi) = \sum_{\chi \in \Omega_{\text{AOL}}} w(\chi)\, \kappa(\chi)^2,$$

where $w(\chi) \ge 0$ is a site weight and Ω_{AOL} is the finite occupied region.

Definition 14 (Load Stress Proxy)**.** *Define an edge load proxy*

$$\sigma(\chi) = \max_{\delta \in \mathcal{N}_6} |\nabla_\delta \psi(\chi)|$$

and define load accumulation

$$\mathcal{S}(\psi) = \sum_{\chi \in \Omega_{\text{AOL}}} w(\chi)\, \sigma(\chi)^2.$$

Proposition 5. *If $\mathcal{K}(\psi)$ or $\mathcal{S}(\psi)$ diverges under routing updates, then no stable locking exists within the current basin.*

Definition 15 (Budgeted EQUI Form)**.** *A computable sufficient form of* EQUI *is*

$$\mathrm{EQUI}(\Phi) = 1 \iff \mathcal{E}(\Phi) \le E_{\max} \wedge \mathcal{K}(\psi) \le K_{\max} \wedge \mathcal{S}(\psi) \le S_{\max} \wedge \kappa_{\max}(\psi) \le \kappa_{\text{crit}} \wedge \sigma_{\max}(\psi) \le \sigma_{\text{crit}}.$$

Phase Alignment Lock (PAL)

Definition 16 (PAL)**.** $\mathrm{PAL} : \Phi \to \{0, 1\}$ *is a temporal coherence operator.* $\mathrm{PAL}(\Phi) = 1$ *if and only if* Φ *remains within an* ϵ*-neighborhood of a phase-consistent manifold for a minimum dwell time* τ*.*

Proposition 6. *PAL prevents commitment of transient, oscillatory, or metastable routing states.*

Proof. Transient configurations do not satisfy the dwell-time requirement and therefore cannot satisfy PAL. They are excluded from locking. □

PAL Dwell-Time Dynamics

Let $\theta(t) \in \mathbb{R}^m$ be a phase descriptor extracted from the routing layer state $\Phi(t)$ by a measurable map

$$\theta(t) = \Theta(\Phi(t)).$$

Let $\mathcal{P} \subset \mathbb{R}^m$ be the phase-consistent manifold.

Definition 17 (Phase Deviation Functional)**.** *Define the phase deviation*

$$\delta_{\mathcal{P}}(t) = \mathrm{dist}\big(\theta(t), \mathcal{P}\big),$$

where dist *is a chosen norm distance in* $\mathbb{R}^m$*.*

Definition 18 (PAL Timer)**.** *Given tolerance* $\epsilon > 0$ *and dwell time* $\tau > 0$*, define the dwell timer* $T(t)$ *by*

$$\frac{dT}{dt} = \begin{cases} 1 & \text{if } \delta_{\mathcal{P}}(t) \le \epsilon, \\ -\gamma T(t) & \text{if } \delta_{\mathcal{P}}(t) > \epsilon, \end{cases} \qquad T(0) = 0,$$

with relaxation constant $\gamma > 0$*.*

Definition 19 (PAL Gate)**.** *Define the PAL gate as*

$$\mathrm{PAL}(\Phi(t)) = 1 \iff T(t) \ge \tau.$$

Proposition 7. *If* $\delta_{\mathcal{P}}(t)$ *intermittently exceeds* ϵ*, then PAL enforces re-stabilization before commitment.*

Proof. When $\delta_{\mathcal{P}}(t) > \epsilon$, the timer decays. Therefore the condition $T(t) \ge \tau$ cannot be met until the state remains phase-consistent for sufficient dwell time. □

Logarithmic Lift and Depth Ordering

Definition 20 (Logarithmic Lift)**.** *Let* $s \in \mathbb{R}_{>0}$ *be a surface scale coordinate. The lift operator* L *is defined by*

$$L(s) = \log(s)$$

and induces a well-founded depth ordering on morphogenic states.

Logarithmic lift is used here as a stratification operator and does not presuppose embedding within a pre-existing spatial manifold.

Proposition 8. *Logarithmic lift stratifies scale and produces discrete depth layers suitable for structural commitment.*

Proof. Exponential separations in surface scale map to additive separations under log, producing bounded, ordered layers with no infinite descending chains. □

Layered Locking and Commitment

Definition 21 (Locking Operator)**.** *A locking operator* Λ *commits an admissible configuration into persistent identity-route structure and removes it from further routing-layer modification.*

Definition 22 (Layered Locking)**.** *Layered locking is the application of* Λ *at successive lift levels.*

Proposition 9. *Layered locking converts coordinate-route states into identity-route structure states.*

Proof. Once Λ is applied, the state is no longer modified by routing-layer updates and therefore persists as an identity-route structure. □

Morphogenic Transition System

A Pattern Field Theory morphogenic system is defined by the operator tuple

$$\mathcal{M} = (\Phi, \Delta, \mathcal{A}, \mathrm{EQUI}, \mathrm{PAL}, L, \Lambda)$$

with evolution rule

$$\Phi_{t+1} = \begin{cases} \Lambda(\Phi_t) & \text{if } \mathcal{A}(\Phi_t) = 1 \land \mathrm{EQUI}(\Phi_t) = 1 \land \mathrm{PAL}(\Phi_t) = 1 \\ \Phi_t + \Delta(\Phi_t) & \text{if } \mathcal{A}(\Phi_t) = 1 \\ \text{forbidden} & \text{otherwise.} \end{cases}$$

This defines a constrained, terminating, computable state transition system.

Structural Closure Theorem

Theorem 1. *Every Pattern Field Theory morphogenic process governed by* $\mathcal{M}$ *terminates in a finite set of identity-route structures embedded on the Allen Orbital Lattice.*

Proof. Admissibility restricts exploration to finite basins. Logarithmic lift defines a well-founded ordering over structural commitment depth, which prevents infinite descending chains of admissible continuation. PAL excludes transient states. EQUI excludes energetically, geometrically, or mechanically unstable states. Λ is irreversible. Therefore infinite routing without commitment is impossible and the process must terminate in a finite set of locked identity-route structures. □

Corollary 1. *Pattern Field Theory morphogenesis is a structure-generating system, not a pattern-generating system.*

QuantaHex Regime

Definition 23 (Admissible Unit and Separation Bound)**.** *Let a locked identity unit be represented by a compact occupied support region $U \subset \Omega_{\mathrm{AOL}}$ with characteristic radius r measured in d_E. Let $c(U) \in \mathbb{Z}^2$ denote its AOL site center. For two units U_i, U_j define their center separation as*

$$d_E^{ij} = d_E(c(U_i), c(U_j)), \qquad d_{\mathrm{AOL}}^{ij} = d_{\mathrm{AOL}}(\widehat{c(U_i)}, \widehat{c(U_j)}).$$

Define a hard separation constraint $d_E^{ij} \geq d_{\min}$ for all nearest-neighbor unit pairs.

Lemma 1 (Metric Compatibility Bound)**.** *For any two AOL sites χ_1, χ_2 one has*

$$\ell\, d_{\mathrm{AOL}}(\chi_1, \chi_2) \leq d_E(\chi_1, \chi_2) \leq \ell\, d_{\mathrm{AOL}}(\chi_1, \chi_2).$$

In particular $d_E = \ell\, d_{\mathrm{AOL}}$ on the AOL site set.

Proof. By construction, each graph step in $\mathcal{N}_6$ maps under ι to a Euclidean step of length ℓ. Any shortest graph path of length d_{AOL} concatenates d_{AOL} such steps and yields Euclidean length ℓd_{AOL}. Since ι is an isometry on the generated lattice edges, the site-to-site Euclidean distance equals ℓd_{AOL}. □

Lemma 2 (Curvature and Load Growth Under Crowding)**.** *Fix ψ representing a locked field on Ω_{AOL}. If a configuration reduces the minimal nearest-neighbor separation among occupied centers while maintaining comparable amplitude scale of ψ, then the maxima $\kappa_{\max}(\psi)$ and $\sigma_{\max}(\psi)$ cannot decrease and generically increase.*

Proof. Reducing separation forces larger discrete gradients $\nabla_\delta \psi$ across fewer lattice steps to accommodate comparable amplitude variation over shorter distance, increasing $\sigma(\chi)$ on at least one site. The Laplacian $\Delta_{\mathrm{AOL}}\psi$ is a sum of neighbor differences, so increased edge differences increase the magnitude of $\Delta_{\mathrm{AOL}}\psi$ on at least one site, increasing $\kappa(\chi)$ and therefore $\kappa_{\max}(\psi)$. Hence $\sigma_{\max}$ and $\kappa_{\max}$ do not decrease and generically increase under crowding. □

Theorem 2 (QuantaHex Maximal Admissible Attractor in 2D Isotropic Basins)**.** *In two-dimensional isotropic basins on the Allen Orbital Lattice, the maximally admissible identity-route structure attractor class is hexagonal.*

Proof. Consider identity units whose centers lie on an infinite periodic set of AOL sites. Under isotropy, admissibility and EQUI depend only on separation statistics and induced curvature-load maxima, not on privileged direction.

Let $\mathcal{L}$ be a periodic center set of fixed asymptotic density. Let $d_{\min}(\mathcal{L})$ be its minimal nearest-neighbor Euclidean separation and let $\mathrm{Var}_1(\mathcal{L})$ be the variance of its nearest-neighbor separations.

A hexagonal (triangular) lattice center set $\mathcal{L}_{\mathrm{hex}}$ uniquely maximizes $d_{\min}(\mathcal{L})$ among Bravais lattices for fixed density and minimizes $\mathrm{Var}_1(\mathcal{L})$ under isotropy. By the separation constraint, the admissible density ceiling is controlled by whether $d_{\min}(\mathcal{L}) \geq d_{\min}$ remains satisfiable.

For any non-hexagonal periodic $\mathcal{L}$ at the same density, either:

$$d_{\min}(\mathcal{L}) < d_{\min}(\mathcal{L}_{\mathrm{hex}}),$$

or nearest-neighbor separations are less uniform so that $\mathrm{Var}_1(\mathcal{L}) > \mathrm{Var}_1(\mathcal{L}_{\mathrm{hex}})$.

In the first case, the separation constraint fails earlier during densification or continuation, violating admissibility before $\mathcal{L}_{\mathrm{hex}}$ does.

In the second case, local crowding occurs in some neighborhood even if the global density is fixed. By the crowding lemma, that increases $\sigma_{\max}(\psi)$ and $\kappa_{\max}(\psi)$, and therefore increases the risk of violating the EQUI bounds

$$\kappa_{\max}(\psi) \le \kappa_{\text{crit}}, \qquad \sigma_{\max}(\psi) \le \sigma_{\text{crit}}$$

and budget bounds

$$\mathcal{K}(\psi) \le K_{\max}, \qquad \mathcal{S}(\psi) \le S_{\max}.$$

Thus the non-hexagonal class violates EQUI earlier than $\mathcal{L}_{\text{hex}}$ under maximal admissible continuation.

Therefore, under isotropic basin continuation constrained by admissibility and EQUI, the maximal admissible density and maximal admissible headroom occurs for the hexagonal class. Hence the identity-route structure attractor class selected under maximal admissible continuation is hexagonal. □

Definition 24 (QuantaHex)**.** *The QuantaHex regime is the hexagonal identity-route structure attractor class on the Allen Orbital Lattice.*

Relation to Continuous Morphogenesis

Turing-type reaction-diffusion systems and other continuous morphogenic models implement only the routing-layer operators (Φ, Δ). They do not implement admissibility, EQUI, PAL, logarithmic lift, or locking. Therefore they remain coordinate-route systems and cannot produce identity-route structure.

When embedded inside the Pattern Field Theory morphogenic operator set, such systems become structurally closed and converge to discrete identity-route structures.

Reaction–Diffusion Containment Statement

A standard two-field reaction–diffusion system is of the form

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u, v), \qquad \frac{\partial v}{\partial t} = D_v \nabla^2 v + g(u, v).$$

This is a coordinate-route routing layer. It is contained in the present framework by taking $\Phi = (u, v)$ and defining $\Delta(\Phi)$ as the discretized diffusion-reaction increment. Structural closure requires adding $\mathcal{A}$, EQUI, PAL, L, and Λ exactly as defined above.

Computability and Replication

Replication requires implementing:

- A continuous routing-layer update operator Δ
- An admissibility predicate $\mathcal{A}$
- The EQUI equilibrium constraint (including budgeted forms)
- The PAL temporal coherence constraint (including dwell-time dynamics)

- A lift level assignment via L
- A commit rule via Λ

The system is simulated as a constrained state machine over a discretized field coupled to an AOL proxy representation.

Repair Dynamics Simulation Pseudocode

The following pseudocode implements routing, admissibility, PAL timing, EQUI budgets, and locking, including regeneration versus fibrosis behavior by basin reachability.

```
Inputs:
  Phi0: initial routing-layer field
  Psi0: initial AOL structure potential (optional)
  A(Phi): admissibility predicate
  EQUI(Phi,Psi): equilibrium operator (energy/curvature/load budgets)
  Theta(Phi): phase descriptor
  dist_to_P(theta): distance to phase manifold P
  epsilon, tau, gamma: PAL parameters
  L: logarithmic lift operator for depth scheduling
  Lambda_lock: locking operator producing identity-route structure on AOL
  BasinID(state): basin classifier (invariant signature)
  target_basin: basin id of pre-damage identity structure (if defined)

State:
  Phi <- Phi0
  Psi <- Psi0
  T <- 0
  locked_structures <- empty list

Loop over time steps t = 1..Tmax:

  # 1) Route update (coordinate-route)
  Phi_candidate <- Phi + Delta(Phi)

  # 2) Admissibility gate
  if A(Phi_candidate) == false:
      Phi <- ProjectToAdmissible(Phi)
      continue

  Phi <- Phi_candidate

  # 3) PAL dwell-time update
  theta <- Theta(Phi)
  if dist_to_P(theta) <= epsilon:
      T <- T + dt
  else:
      T <- max(0, T - gamma*T*dt)

  # 4) Optional: update AOL proxy fields from Phi (for EQUI)
  Psi <- UpdatePsiFromPhi(Phi, Psi)

  # 5) Basin tracking (for repair classification)
  basin_now <- BasinID(Phi, Psi)
```

```
  # 6) Commit condition (identity-route structure)
  commit_ok <- (T >= tau)

  # 7) EQUI gate
  if commit_ok and EQUI(Phi, Psi) == true:
      S <- Lambda_lock(Phi, Psi, depth=L(scale(Phi)))
      locked_structures.append(S)

      # repair classification
      if target_basin is defined:
          if basin_now == target_basin:
              Tag(S, "regeneration")
          else:
              Tag(S, "fibrosis")

      Phi <- ResidualFieldAfterLock(Phi, S)
      T <- 0

End loop

Outputs:
  locked_structures, with tags regeneration/fibrosis when target_basin is defined
```

Regeneration and Fibrosis as Structural Outcomes

Definition 25 (Regeneration)**.** *Regeneration is the restoration of a prior identity-route structure by routing-layer exploration that remains within the same admissible basin and satisfies* EQUI *and* PAL *constraints at commitment.*

Definition 26 (Fibrosis)**.** *Fibrosis is the formation of an alternative identity-route structure when the original basin is no longer reachable under admissibility constraints.*

Proposition 10. *If a damaged structure admits a routing-layer path back into its original basin while satisfying* EQUI *and* PAL*, then regeneration occurs. Otherwise, the system converges to a different basin and locks a replacement structure.*

Proof. Admissibility restricts reachable configurations. If the original basin remains reachable, layered locking can re-commit the original structure. If not, routing necessarily converges to another basin minimum and locks a different structure, which is observed as fibrosis. □

Scope of Structural Results

The results presented here establish operator-level completion of morphogenic evolution under admissibility and logarithmic stratification.

They define conditions for: (i) termination of exploratory routing, (ii) selection of stable identity-route structures, and (iii) emergence of maximal admissible packing regimes under basin continuation.

These results characterize structural closure properties of pattern-forming systems independent of specific physical realization, and therefore apply to biological, chemical, and abstract dynamical morphogenesis models.

Conclusion

Pattern Field Theory morphogenesis is a complete field-to-structure transition system. Admissibility, EQUI, PAL, logarithmic lift, and layered locking close the structural gap of continuous morphogenesis and produce discrete, persistent identity-route structures on the Allen Orbital Lattice. Turing-type reaction–diffusion morphogenesis is contained as coordinate-route dynamics that becomes structurally closed under the PFT operator set.

Glossary

Admissibility A predicate constraint $A(\Phi)$ restricting routing-layer evolution to configurations that satisfy structural, energetic, curvature, and basin constraints.

Basin A maximal connected region of state space closed under admissible transitions. Basins define structural continuation domains for morphogenic evolution.

Routing Layer A continuous morphogenic field Φ whose evolution explores configuration space without structural commitment. Routing layers implement coordinate-route dynamics but do not define persistence.

Coordinate Route A morphogenic mode in which system state is fully determined by its position in configuration space and remains continuously modifiable by routing-layer updates.

Identity-Route Structure A morphogenic configuration committed by the locking operator Λ and removed from routing-layer modification. Identity-route structures persist and define discrete structural objects.

Allen Orbital Lattice (AOL) The discrete structural substrate on which identity-route structures are embedded, represented by axial or cube integer coordinate charts with six nearest-neighbor adjacency.

EQUI Operator The equilibrium admissibility operator $\mathrm{EQUI}(\Phi)$ enforcing bounded energy, curvature, and structural load constraints prior to locking.

PAL (Phase Alignment Lock) A temporal coherence operator $\mathrm{PAL}(\Phi)$ requiring dwell-time stability within a phase-consistent manifold before structural commitment.

Logarithmic Lift A structural stratification operator

$$L(s) = \log(s), \quad s \in \mathbb{R}_{>0}$$

which induces a well-founded depth ordering over morphogenic scale.

Layered Locking Successive application of the locking operator Λ across depth layers defined by logarithmic lift.

Locking Operator The operator Λ that commits an admissible configuration into a persistent identity-route structure and removes it from routing evolution.

Curvature Proxy A discrete Laplacian-derived scalar $\kappa(\chi)$ representing local curvature accumulation on the Allen Orbital Lattice.

Load Proxy A discrete gradient-derived scalar $\sigma(\chi)$ representing local structural stress accumulation on the Allen Orbital Lattice.

QuantaHex Regime The hexagonal identity-route structure attractor class selected under isotropic admissible continuation on the Allen Orbital Lattice.

Regeneration Restoration of a prior identity-route structure when routing remains within the original admissible basin and satisfies commitment constraints.

Fibrosis Formation of an alternative identity-route structure when the original basin is no longer reachable under admissibility constraints.

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Document Timestamp and Provenance

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