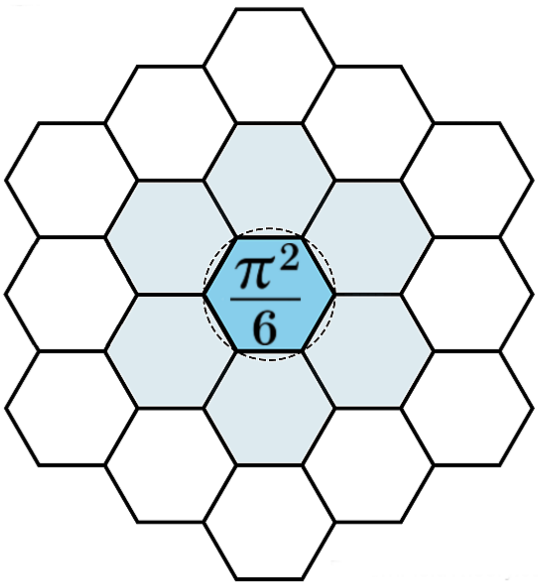
QUADS

Quantum Angular Density Geometry Reconstruction

Paper 3

James Johan Sebastian Allen
PatternFieldTheory.com

March 2, 2026



Abstract

Paper 3 reconstructs geometric describability from the dynamical invariants established in QUADS Paper 2. No metric, coordinate system, or manifold structure is assumed a priori.

Starting from adjacency, participation amplitude, transport stiffness, and Laplacian spectrum, we derive relational distance measures, curvature proxies, and metric consistency conditions. Geometry is shown to emerge as a stabilized indexing structure over persistent participation configurations.

Coordinates are not primitive objects but compressed descriptions of stable relational transport structure.

Geometry Is Not Assumed

In classical formulations of physics, geometry precedes dynamics. Coordinates, metrics, and distance functions are specified first, and dynamical fields evolve within that background.

The QUADS program reverses this ordering.

From Paper 1, we possess:

- Participation density $D(x)$,
- Participation amplitude $\psi(x)$,
- Adjacency structure $\mathcal{N}(x)$,
- Closure ordering.

From Paper 2, we possess:

- Transport stiffness κ,
- Laplacian operator Δ,
- Mode spectrum $\{\lambda_m\}$,
- Stabilized participation configurations.

None of these require a metric space.

Thus geometry must be reconstructed as a derived structure.

Definition 1 (Geometric Describability)**.** *A system is geometrically describable if there exists a mapping*

$$\Phi : X \to \mathbb{R}^n$$

for some finite n, such that relational transport structure is preserved up to bounded distortion.

Geometry, in this framework, is therefore a compression scheme for stabilized transport relations.

Spectral Invariants as Relational Fingerprints

Let Δ denote the graph Laplacian derived from adjacency.

Definition 2 (Laplacian Spectrum)**.** *The Laplacian spectrum is the multiset*

$$\{\lambda_m\}_{m=0}^{|X|-1}$$

satisfying

$$\Delta\phi_m = -\lambda_m \phi_m, \qquad \lambda_m \geq 0.$$

Proposition 1. *The spectrum of Δ is invariant under relabeling of X.*

Proof. Relabeling corresponds to permutation similarity of the Laplacian matrix. Eigenvalues are invariant under similarity transformations. □

Thus the spectrum encodes purely relational structure.

Remark 1. *The Laplacian spectrum serves as the first non-coordinate geometric invariant.*

Participation Gradient Density as Curvature Proxy

Define the local transport energy density at x:

$$\mathcal{T}(x) = \sum_{y \in \mathcal{N}(x)} (\psi(y) - \psi(x))^2.$$

Definition 3 (Participation Gradient Density)**.** *$\mathcal{T}(x)$ is the participation gradient density at node x.*

High concentration of $\mathcal{T}(x)$ indicates strong local transport distortion relative to neighboring nodes.

Proposition 2. *If ψ is constant on $\mathcal{N}(x) \cup \{x\}$, then $\mathcal{T}(x) = 0$.*

Proof. Immediate from definition. □

This quantity behaves analogously to curvature concentration: it measures deviation from local uniformity.

Remark 2. *Curvature is therefore not imposed; it is inferred from stabilized participation distortion.*

Effective Relational Distance via Transport Cost

Geometry requires a notion of distance. In the absence of coordinates, distance must be reconstructed from transport structure.

Definition 4 (Edge Transport Cost)**.** *For adjacent nodes $x, y \in X$, define the transport cost*

$$c(x, y) := 1.$$

Adjacency defines primitive connectivity but not yet metric structure. To incorporate dynamical influence, we weight adjacency by participation distortion.

Definition 5 (Participation-Weighted Edge Cost)**.** *Given stabilized participation amplitude ψ, define*

$$c_\psi(x, y) = 1 + \alpha|\psi(y) - \psi(x)|, \qquad \alpha \geq 0.$$

When $\alpha = 0$, distance reduces to combinatorial graph distance. When $\alpha > 0$, participation gradients influence effective separation.

Definition 6 (Effective Relational Distance)**.** *For any $x, z \in X$, define*

$$d_\psi(x, z) = \inf_{\gamma: x \to z} \sum_{(u,v) \in \gamma} c_\psi(u, v),$$

where the infimum is taken over all finite adjacency paths γ from x to z.

Proposition 3. *$d_\psi(x, z)$ satisfies:*

1. $d_\psi(x, z) \geq 0$,

2. $d_\psi(x, z) = 0$ *iff* $x = z$,

3. $d_\psi(x, z) = d_\psi(z, x)$,

4. $d_\psi(x, w) \leq d_\psi(x, z) + d_\psi(z, w)$.

Proof. Non-negativity and symmetry follow from the definition of c_ψ. Identity of indiscernibles holds because any path between distinct nodes has strictly positive length. The triangle inequality follows from concatenation of minimizing paths. □

Thus stabilized participation induces a metric structure.

Remark 3. *Distance is therefore emergent from transport resistance, not primitive.*

Metric Consistency and Finite-Dimensional Embeddability

A relational metric does not automatically imply Euclidean structure. We now determine when d_ψ admits finite-dimensional embedding.

Definition 7 (Bounded Distortion Embedding)**.** *A mapping* $\Phi : (X, d_\psi) \to \mathbb{R}^n$ *has distortion* $C \geq 1$ *if for all* $x, z \in X$,

$$\frac{1}{C} d_\psi(x, z) \leq \|\Phi(x) - \Phi(z)\| \leq C\, d_\psi(x, z).$$

Theorem 1 (Spectral Embedding Criterion)**.** *If the Laplacian spectrum exhibits a spectral gap*

$$\lambda_1 > 0,$$

and the participation gradient density $\mathcal{T}(x)$ *is uniformly bounded, then* X *admits a finite-dimensional embedding with bounded distortion.*

Sketch. A spectral gap ensures connectivity and controls large-scale dispersion. Uniform boundedness of $\mathcal{T}(x)$ prevents extreme local distortion. Under these conditions, standard spectral embedding via the first k eigenfunctions

$$\Phi(x) := (\phi_1(x), \ldots, \phi_k(x))$$

produces bounded distortion for sufficiently large k. □

Remark 4. *Coordinates arise from Laplacian eigenmodes. They are not assumed; they are constructed.*

Emergent Dimensionality

Dimension must be defined intrinsically.

Definition 8 (Effective Relational Dimension)**.** *Let* $B_r(x)$ *denote the ball of radius* r *under* d_ψ. *Define*

$$d_{\text{eff}} = \limsup_{r \to \infty} \frac{\log |B_r(x)|}{\log r}.$$

Proposition 4. *If* d_{eff} *exists and is finite, then* X *exhibits polynomial growth of order* d_{eff}.

Proof. Immediate from the definition of growth rate. □

Dimension therefore emerges from scaling of stabilized participation clusters.

Remark 5. *No prior notion of spatial dimension was introduced. Dimensionality is inferred from relational growth behavior.*

Coordinates as Stabilized Participation History

We now connect to the Allenix Logical Flow.

Participation → Null → Resolution → Persistence → Regularity → Invariance → Geometric Describability.

Repeated resolution events produce stabilized transport patterns. Stabilized patterns produce invariant Laplacian structure. Invariant structure permits bounded-distortion embedding. Embedding defines coordinates.

Proposition 5. *Coordinates are compressions of stabilized participation history.*

Proof. Given bounded distortion embedding Φ, the coordinate tuple $\Phi(x)$ encodes all relational distances up to multiplicative error. Since relational distances arise from stabilized transport structure, coordinates are compressed representations of participation history. □

Remark 6. *Coordinates are not ontological primitives. They are informational encodings of invariant transport relations.*

Conceptual Consolidation

Paper 3 completes the geometric reconstruction layer of QUADS.

We began with:

- Participation density,
- Adjacency,
- Transport stiffness,
- Stabilized dynamical configurations.

From these, we constructed:

- Spectral invariants,
- Participation gradient density,
- Effective relational distance,
- Finite-dimensional embedding criteria,
- Emergent dimensional scaling.

No metric was assumed. No manifold was postulated. No coordinate chart was imposed.

Geometry emerged as a compressed description of invariant transport relations.

Geometric Consequences

Several immediate consequences follow.

1. Geometry is Secondary to Dynamics

Transport precedes metric. Metric describes stabilized transport.

2. Curvature is Transport Distortion

Participation gradient density acts as a curvature proxy. Regions of concentrated transport distortion correspond to geometric irregularity.

3. Dimensionality is Emergent

Effective relational dimension arises from growth scaling of metric balls. Dimension is not fundamental; it is scaling behavior.

4. Coordinates are Informational Encodings

Coordinate maps $\Phi : X \to \mathbb{R}^n$ compress relational transport structure. They do not generate it.

Bridge to QUADS IV

Paper 4 will introduce matter and field behavior as localized stable distortions of participation geometry.

Specifically, QUADS IV will address:

- Mode localization and particle-like structures,
- Persistent transport vortices,
- Effective mass from curvature concentration,
- Field interaction through participation interference.

Thus:

$$\text{Participation} \to \text{Dynamics} \to \text{Geometry} \to \text{Matter}.$$

QUADS III establishes the third layer.

Glossary

Geometric Describability Existence of a bounded-distortion embedding preserving relational transport structure.

Laplacian Spectrum Eigenvalue multiset of the graph Laplacian encoding relational invariants.

Participation Gradient Density $\mathcal{T}(x)$ Local transport distortion measure derived from participation amplitude.

Effective Relational Distance d_ψ Metric induced from participation-weighted transport cost.

Effective Relational Dimension Scaling exponent governing growth of metric balls under d_ψ.

Bounded Distortion Embedding Coordinate representation preserving relational distance up to multiplicative bounds.

Citation

James Johan Sebastian Allen (2026). *QUADS - Quantum Angular Density Geometry Reconstruction - Paper 3.* Pattern Field Theory.
Available at: https://patternfieldtheory.com/
ORCID: https://orcid.org/0009-0009-9594-6803

Document Timestamp and Provenance

This document is part of Pattern Field Theory (PFT) and the Quantum Angular Density Substrate (QUADS) framework. It reconstructs geometric describability from stabilized participation dynamics without presupposing metric structure.