Standard Model from Cascade Branching: Emergent QFT on the Allen Orbital Lattice

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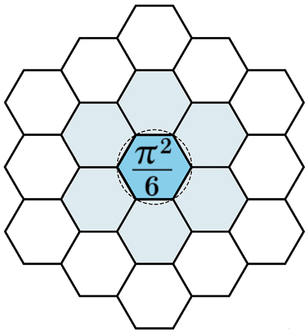

Abstract

Pattern Field Theory (PFT) describes all physical and informational structure as patterns evolving on the Allen Orbital Lattice (AOL), a prime-indexed orbital curvature lattice. Phase Alignment Lock (PAL) enforces discrete flux neutrality on prime-labeled faces and constrains allowed event cascades.

In this paper, quantum field theory (QFT), in particular the Standard Model of particle physics, is recovered as the low-energy, large-scale limit of PAL-constrained cascade branching on the AOL. The color gauge group $SU(3)_c$ emerges from three-prong AOL vertices, the electroweak gauge group $SU(2)_L \times U(1)_Y$ from duplex prime symmetry and PAL-compatible phase winding, and three fermion generations from $\sqrt{n}$ ghost layers in the Lagrange-hex projection. Scattering amplitudes arise as PAL-weighted sums over cascade decay trees, and Cabibbo–Kobayashi–Maskawa (CKM) mixing angles are encoded in PAL phase differences between ghost layers. Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing in the lepton sector follows by the same mechanism.

We also review the historical unification problems that have plagued mainstream physics—non-renormalizable gravity, arbitrary gauge and flavor structure, unexplained mass hierarchies, vacuum energy divergences, and background dependence—and explain how PFT replaces each with a single geometric and combinatorial mechanism on the Allen Orbital Lattice. The Standard Model then appears as an infrared (IR) effective description of PAL-constrained cascades rather than an independent axiom system.

1 Introduction

The Standard Model of particle physics and general relativity (GR) are two of the most successful theories in modern physics. The Standard Model is a quantum field theory (QFT) with gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y,$$

describing strong, weak, and electromagnetic interactions among fermions and gauge bosons. General relativity describes spacetime curvature and gravity through the Einstein field equations.

Despite their empirical success, these theories are structurally disjoint. Gravity resists renormalization in standard QFT frameworks, the Standard Model gauge group and fermion content appear arbitrary, and no widely accepted microscopic origin for spacetime exists. Attempts to quantize gravity or to embed the Standard Model into larger groups have not produced a simple, universally accepted unification.

Pattern Field Theory (PFT) takes a different route. Instead of quantizing an already continuous spacetime, PFT starts from a discrete orbital-curvature lattice—the Allen Orbital Lattice (AOL)—on which patterns evolve. The lattice is prime-indexed and carries curvature weights, phases, and recursion depth. Phase Alignment Lock (PAL) enforces flux neutrality on prime-labeled faces; cascades are allowed only when PAL is respected at every branching.

In this framework:

- The lattice (AOL) is the underlying combinatorial and geometric substrate.
- PAL is the coherence rule constraining evolution.
- Event cascades encode dynamics and interaction histories.
- Continuum field theories, including QFT and GR, are infrared (IR) projections of these discrete structures.

This paper focuses on the emergence of the Standard Model from PAL-constrained cascades. We show that gauge groups, fermion families, mixing matrices, and scattering amplitudes can be interpreted as shadows of AOL topology and PAL phases, and we connect this construction to long-standing unification problems.

2 Historical Unification Problems

2.1 GR vs QFT incompatibility

General relativity is a classical field theory of spacetime curvature. Quantum field theory is built on a fixed background spacetime. Naively quantizing the metric as another field leads to non-renormalizable divergences at high energy scales: the gravitational coupling has negative mass dimension, and loop corrections introduce infinitely many counterterms.

2.2 Arbitrary Standard Model gauge structure

The Standard Model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is chosen because it fits observations. There is no derivation of this exact product structure from deeper combinatorial or geometric principles in conventional QFT. Grand unification theories embed this group in larger ones but must then explain symmetry breaking patterns and the absence of additional predicted particles.

2.3 Three generations and flavor puzzles

The Standard Model contains three generations of quarks and leptons with identical gauge charges but different masses and mixing patterns. The existence of exactly three generations is unexplained. The pattern of Yukawa couplings, which control masses and mixings, appears arbitrary. CKM and PMNS matrices contain angles and phases not fixed by the theory.

2.4 Hierarchy and naturalness

The large separation of scales between electroweak physics and Planck-scale physics creates the hierarchy problem: quantum corrections to scalar masses are sensitive to far higher scales, requiring delicate cancellations. The cosmological constant problem is a related extreme sensitivity of vacuum energy to ultraviolet (UV) physics.

2.5 Vacuum energy and divergences

Standard QFT calculations produce infinite vacuum energies and UV divergences that must be regularized and renormalized. The procedure works for renormalizable gauge theories but lacks a finite combinatorial origin. Gravity resists this treatment entirely.

2.6 Background dependence

QFT normally assumes a fixed background metric. Even when this metric is curved, it is treated as externally supplied rather than dynamically induced by the same quantum fluxes. This background dependence conflicts with the spirit of GR, where geometry is fully dynamic.

Any unifying framework must replace these issues with structural mechanisms rather than new free parameters.

3 PAL-Constrained Cascades on the AOL

3.1 Allen Orbital Lattice

The Allen Orbital Lattice (AOL) is a discrete orbital-curvature lattice with:

- sites x carrying local pattern states,
- edges $(x, x + \hat{\mu})$ with curvature weights and phase increments,
- faces S_p labeled by primes p with oriented boundaries ∂S_p,
- higher cells encoding recursion and cascades.

Curvature is encoded via plaquette phases around faces:

$$F(\partial S_p) = \sum_{e \in \partial S_p} \omega(e),$$

where $\omega(e)$ is an edge contribution.

3.2 Phase Alignment Lock

Definition 1 (Phase Alignment Lock (PAL))**.** *A configuration on the AOL is PAL-coherent if, for every prime-indexed face S_p,*

$$\nabla \cdot F(\partial S_p) = 0, \tag{1}$$

where $\nabla\cdot$ is the discrete divergence operator. PAL enforces exact flux neutrality on all prime-labeled faces.

PAL forbids net flux leakage through any prime-indexed 2-cell. Conserved quantities such as charge, color flow, and various quantum numbers are manifestations of PAL-constrained flux in different sectors.

3.3 Event cascades

A *cascade* is a PAL-consistent branching process

$$\phi_0 \to \phi_1 + \phi_2 + \cdots ,$$

where ϕ_0 is an initial PAL-stable configuration and ϕ_i are descendants. Each branching occurs at lattice vertices and must obey PAL on all faces involved. A cascade can be represented as a rooted tree $\mathcal{T}$ embedded in the AOL, with vertices labeled by prime indices p_v and PAL phases θ_v.

4 Gauge Structure from AOL Topology

4.1 Color $SU(3)_c$ from three-prong vertices

Consider AOL vertices where three prime-indexed directions meet coherently. Let the outgoing branches carry flux vectors $\boldsymbol{F}_1, \boldsymbol{F}_2, \boldsymbol{F}_3$. PAL neutrality imposes:

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 0.$$

Theorem 1 (Color gauge emergence)**.** *Three-prong PAL-coherent vertices on the AOL realise the color gauge group $SU(3)_c$ in the infrared limit.*

Sketch. The constraint $\boldsymbol{F}_1+\boldsymbol{F}_2+\boldsymbol{F}_3 = 0$ restricts allowed flux assignments on the three branches. Modulo overall scaling and phase, the space of assignments has the structure of a complex triplet, and local transformations mixing the branches while preserving PAL form a group isomorphic to $SU(3)$ acting on this internal color space. In the continuum limit, PAL-preserving connections that redistribute flux among the branches appear as gluon fields of quantum chromodynamics. □

4.2 Electroweak $SU(2)_L \times U(1)_Y$ from duplex symmetry

For the electroweak sector, we focus on duplex prime symmetries: pairs of directions (p, q) such that exchanging p and q plus an allowed PAL phase shift leaves all constraints intact. This defines a two-component internal structure.

Theorem 2 (Electroweak emergence)**.** *Duplex prime symmetry on the AOL produces a doublet structure realising $SU(2)_L$, while PAL-compatible phase winding around closed loops yields a $U(1)_Y$ factor. Together these define an emergent $SU(2)_L \times U(1)_Y$ electroweak gauge structure.*

Sketch. The pair (p, q) defines a two-component object, analogous to a weak isospin doublet, with PAL-preserving transformations mixing the components. These transformations form an $SU(2)$ symmetry. Meanwhile, PAL constraints on phases accumulated around loops quantise an Abelian phase, producing an effective $U(1)$ associated with hypercharge. The combined internal symmetry matches the electroweak gauge group in the infrared description. □

5 Fermion Generations and $\sqrt{n}$ Ghost Layers

5.1 Lagrange-hex projection

The Lagrange-hex projection organises minimal displacement modes of the AOL into classes at distances $\sqrt{n}$, with n a positive integer. The first few classes $\sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots$ define "ghost layers": structured patterns of permitted moves and interactions at fixed displacement scales.

5.2 Three generations from three layers

The Standard Model has three generations of fermions. In PFT, these arise from three dominant ghost layers $n = 1, 2, 3$.

Theorem 3 (Fermion generations from ghost layers)**.** *Three fermion generations in the Standard Model correspond to three dominant* $\sqrt{n}$ *ghost layers (*$n = 1, 2, 3$*) in the Lagrange-hex projection of the AOL.*

Sketch. PAL-constrained cascades explore the lattice through sequences of minimal moves. Different ghost layers correspond to different displacement scales and curvature couplings. Cascades mostly confined to the $\sqrt{1}$ layer yield one family of effective fermions with characteristic mass scale, while those involving $\sqrt{2}$ and $\sqrt{3}$ yield additional families with higher scales. Approximate scaling

$$m_f^{(n)} \propto \sqrt{n}\, m_0$$

captures the trend, where m_0 is a base scale set by local curvature and PAL. Thus three structurally distinct ghost layers map to the three observed generations. □

This ties flavor structure to AOL geometry rather than free parameters.

6 Scattering and Mixing from Cascades

6.1 Cascade decay trees as scattering diagrams

In QFT, scattering amplitudes are computed as sums over Feynman diagrams. In PFT, PAL-consistent cascade trees play a similar role.

Let $\mathcal{T}$ be the set of PAL-admissible cascade trees connecting a given initial configuration to a given final configuration. The amplitude is

$$\mathcal{A} = \sum_{T \in \mathcal{T}} W(T), \tag{2}$$

where $W(T)$ is a weight associated with tree T. A structurally motivated choice is

$$W(T) = \prod_{v \in T} \frac{1}{p_v^2} e^{i\theta_v}, \tag{3}$$

with p_v the prime index at vertex v and θ_v the PAL phase there.

6.2 CKM and PMNS mixing from PAL phases

Different cascade trees connecting the same flavor channels may traverse different combinations of ghost layers and prime indices, accumulating different total PAL phases

$$\Theta(T) = \sum_{v \in T} \theta_v.$$

Interference depends on phase differences and the pattern of $1/p_v^2$ factors. Grouping contributions by flavor yields a unitary matrix structure.

Theorem 4 (Emergent CKM/PMNS mixing)**.** *If branching weights obey* $\Gamma_v \propto 1/p_v^2$ *and PAL phases depend on ghost layers, then interference among PAL-consistent cascade trees yields effective CKM- and PMNS-like mixing matrices with angles set by AOL PAL phase differences between layers.*

Sketch. Summing over trees for each flavor transition gives complex amplitudes whose phases and magnitudes depend on which primes and ghost layers appear along the paths. The overlaps define a unitary mixing matrix relating flavor and propagation eigenstates. Asymmetries in PAL phase assignments across layers and primes produce complex phases responsible for CP violation, analogous to CKM and PMNS phases. □

7 PFT Resolution of Unification Problems

7.1 GR and QFT from one substrate

In PFT, spacetime geometry (GR sector) and matter/gauge sectors (QFT sector) both emerge from AOL and PAL. There is no separate quantization of an independent metric; both are IR projections of one discrete combinatorial substrate. Non-renormalizability of gravity is replaced by finite combinatorics and PAL constraints.

7.2 Gauge structure from AOL geometry

The Standard Model gauge group is not imposed by hand but arises from:

- three-prong vertices (color $SU(3)_c$),
- duplex prime symmetries ($SU(2)_L$),
- PAL loop phases ($U(1)_Y$).

The product structure reflects factorisation properties of AOL connectivity and PAL, not a free choice.

7.3 Generations, masses, and mixings

Three generations correspond to three ghost layers. Mass hierarchies follow from layer-dependent curvature couplings and approximate $\sqrt{n}$ scaling, while mixing matrices arise from interference among cascade trees across layers. Yukawa matrices are effective summaries of deeper combinatorial structure rather than arbitrary input parameters.

7.4 Hierarchy, vacuum energy, and divergences

In PFT, scales come from lattice spacing, curvature assignments, prime distributions, and PAL patterns. There are no continuous bare parameters that must be tuned across many orders of magnitude. PAL cuts off many configurations that would correspond to divergent continuum integrals, and scattering amplitudes are finite sums or convergent series over cascade trees weighted by $1/p^2$ factors. Vacuum energy is determined by PAL-coherent configurations rather than integration over unbounded mode densities.

7.5 Background independence

Both geometry and fields arise from AOL and PAL; there is no fixed background. Different effective geometries correspond to different global PAL-coherent configurations. Background independence is restored in a combinatorial sense.

8 Discussion and Outlook

We have described how the Standard Model can be interpreted as an emergent QFT from PAL-constrained cascades on the Allen Orbital Lattice. Gauge groups, fermion families, mixing matrices, and scattering amplitudes become geometric and combinatorial features of AOL and PAL rather than independent axioms.

From this perspective:

- GR and QFT are two IR faces of one Pattern Field Theory.
- The Standard Model gauge structure and flavor content emerge from AOL topology.
- Hierarchy and naturalness problems are reframed as questions about lattice geometry and PAL.

Further work includes:

- quantitative extraction of masses and mixing angles from explicit AOL configurations,
- extension to neutrino phenomenology and possible sterile states,
- analysis of PFT signatures in high-energy experiments.

Appendix A — Glossary of Terms and Acronyms

Pattern Field Theory (PFT) Unified framework in which all structure and dynamics are described as patterns evolving on the Allen Orbital Lattice.

Allen Orbital Lattice (AOL) Prime-indexed orbital-curvature lattice carrying sites, edges, faces, curvature weights, phase data, and recursion structure.

Phase Alignment Lock (PAL) Coherence condition requiring exact flux neutrality on all prime-indexed faces of the AOL.

Cross-Coherent Cascade Theory (CCCT) Branch of PFT describing cascades and coherence collapse across coupled networks and domains.

Quantum Field Theory (QFT) Quantum theory describing particles and interactions as excitations of fields; in PFT, an infrared effective description of PAL-constrained cascades.

Standard Model Quantum field theory of elementary particles based on gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with three fermion generations.

$SU(3)_c$ Non-Abelian gauge group of the strong interaction (color).

$SU(2)_L$ Non-Abelian gauge group of the weak interaction acting on left-handed components.

$U(1)_Y$ Abelian gauge group associated with hypercharge Y.

CKM Matrix Cabibbo–Kobayashi–Maskawa matrix describing quark flavor mixing.

PMNS Matrix Pontecorvo–Maki–Nakagawa–Sakata matrix describing neutrino mixing.

IR (Infrared) Limit Low-energy, large-distance limit of a theory.

UV (Ultraviolet) High-energy, short-distance regime where continuum QFT typically exhibits divergences.

Ghost Layer Structured pattern of allowed lattice moves at fixed displacement scale $\sqrt{n}$ in the Lagrange-hex projection.

Lagrange-hex Projection Representation of the AOL that organises minimal displacement modes into hexagonally structured layers labelled by distances $\sqrt{n}$.

Cascade Tree Graph of PAL-consistent branching events from an initial configuration to a set of descendants.

Appendix B — PFT Internal Bibliography

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Document Timestamp and Provenance

This paper forms part of the dated Pattern Field Theory research chain beginning May 2025, recorded through server logs, cryptographic hashes, and versioned texts on PatternFieldTheory.com, establishing priority and authorship continuity for the emergent QFT construction on the Allen Orbital Lattice.