Pauli Exclusion as Geometric Occupancy on the Allen Orbital Lattice

Duplex Phase Opposition as the Stable Coherence State of Fermions

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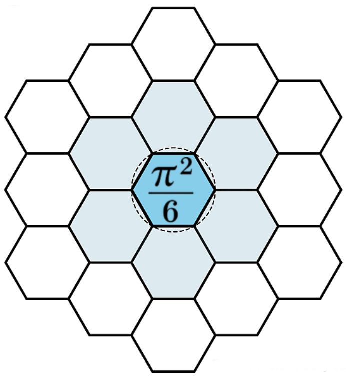

Abstract

The Pauli Exclusion Principle is presented as a geometric occupancy rule of the Allen Orbital Lattice (AOL). An electron is modeled as a bounded coherence packet propagating on curvature-minimizing hexagonal rings $\mathcal{H}_k$. Each spatial orbital corresponds to a ring mode (k, m) with a well-defined duplex internal phase (ϕ_E, ϕ_B). A local curvature energy $\mathcal{E}$ is minimized when two packets in the same mode occupy duplex phases separated by π. Any third packet forces phase conflict, raising $\mathcal{E}$ above the stability threshold and ejecting one packet to another mode. This yields Pauli Exclusion directly from field geometry, without subtractive antisymmetry postulates. The duplex phase map recovers standard spin quantum numbers and predicts measurable phase-separated occupancy patterns in quantum dot and moiré systems.

1 Introduction

Scope. This paper concerns fermionic orbital occupancy and spin pairing as consequences of curvature coherence on the Allen Orbital Lattice.

Allen Orbital Lattice

The Allen Orbital Lattice (AOL) is a discrete hexagonal curvature field that defines stable radial shells and angular sectors. Each shell is indexed by a prime-sequence radius s_n, and each allowed electron orbital corresponds to a mode supported on a specific shell. The lattice does not describe particle positions in space; it defines the geometric constraints under which coherent field packets form stable orbitals, layer spacings, and faceting boundaries.

Pauli Exclusion is presented as a stable occupancy condition of duplex phases on the AOL. The exclusion rule follows from curvature minimization and packet interference on a discrete ring substrate.

2 Mechanism: How Exclusion Arises

1. The AOL is a set of curvature-minimizing hexagonal rings $\mathcal{H}_k$ indexed by shell number k.
2. An electron is represented as a coherence packet ψ with duplex internal phase (ϕ_E, ϕ_B).
3. A spatial orbital is a ring standing mode (k, m) with wavenumber m.
4. Co-occupancy modifies curvature energy

$$\mathcal{E}[\rho, \Delta\theta] = \int_{\mathcal{H}_k} \left(\alpha|\nabla\kappa|^2 + \beta\rho^2 + \gamma\rho^2 \cos\Delta\theta\right) ds.$$

5. Energy is minimized at duplex anti-phase $\Delta\theta = \pi$. Three packets cannot maintain full pairwise anti-phase. One packet transitions to another mode to restore curvature stability.

3 Formal Statement

Definition. Duplex phase $\theta \in S^1$ defines spin label

$$S_z = \frac{\hbar}{2}\mathrm{sgn}(\sin\theta).$$

Proposition. A mode (k, m) supports at most two fermionic packets. If two occupy the mode, they exist with duplex phase separation π. A third increases $\mathcal{E}$ above the curvature threshold and moves to a different (k', m').

4 Mapping to Standard Quantum Numbers

$$(n, \ell, m_\ell, m_s) \longleftrightarrow (k(n, \ell),\, m = m_\ell,\, \theta_{m_s}), \quad \theta_{+\frac{1}{2}} = \theta_0, \quad \theta_{-\frac{1}{2}} = \theta_0 + \pi.$$

Scope and Limitations

This treatment addresses orbital occupancy and spin pairing as geometric phase conditions on the Allen Orbital Lattice. It does not attempt to model Coulomb interactions, exchange correlation functionals, or temperature-dependent many-body effects. These phenomena operate within the occupancy rules established here and may be reintroduced as secondary energy terms when needed.

Significance and Theoretical Consolidation

First-principles derivation of Pauli Exclusion.

Exclusion arises as a geometric occupancy limit on curvature-stable ring modes of the Allen Orbital Lattice. Duplex anti-phase occupancy is the only energy-stable configuration for two fermionic packets in the same orbital mode. A third packet cannot satisfy the curvature stability condition and relocates to another mode.

- Spin is an encoded duplex phase degree of freedom.
- Pairing corresponds to a fixed π phase offset between duplex channels.
- Exclusion follows from the minimum of the curvature energy functional, not from imposed antisymmetry.

Implications across domains.

Disciplinary Impact

Quantum Mechanics
Pauli Exclusion follows from a curvature stability condition on the lattice. The spin state is a duplex phase state tied directly to geometric occupancy.

Condensed Matter
The model predicts discrete occupancy transitions in moiré superlattices and quantum dots, including regimes with screened Coulomb interaction.

Superconductivity
Cooper pairing is duplex anti-phase alignment across neighboring rings. Critical current is set by the inter-ring phase registry.

Quantum Information
The duplex phase is a physical parameter for qubit spin state control. This allows direct geometric state preparation.

Mathematical Physics
Antisymmetry arises from phase spacing geometry on the lattice. Exclusion is a derived stability rule instead of an assumed axiom.

5 Predictions and Tests

- Quantum dots: triply-occupied same-orbital states collapse by discrete mode hopping.
- STM phase imaging: co-occupied states show anti-phase separation on hex substrates.
- Superconductors: Cooper pairing corresponds to duplex anti-phase across neighboring rings.

6 Relation to Spinors

Duplex rotations generate Pauli operators. Two duplex channels form a 2-component spinor Ψ:

$$i\hbar\,\partial_t\Psi = \left(v\,\boldsymbol{\sigma}\cdot\mathbf{p}_{\parallel} + m^* v^2\sigma_z + V_\kappa(\mathcal{H}_k)\right)\Psi.$$

7 Discussion

Pauli Exclusion arises as a geometric occupancy rule of ring modes on the AOL. Spin is phase. Antisymmetry is encoded as duplex phase separation. Orbital structure and pairing emerge as curvature-stable configurations.

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