SRR as a Renormalization Group on the Allen Orbital Lattice

A Coarse-Graining Calculus for Boundary Emergence and Effective Laws

Structural Regime Resolution Series — Paper VIII

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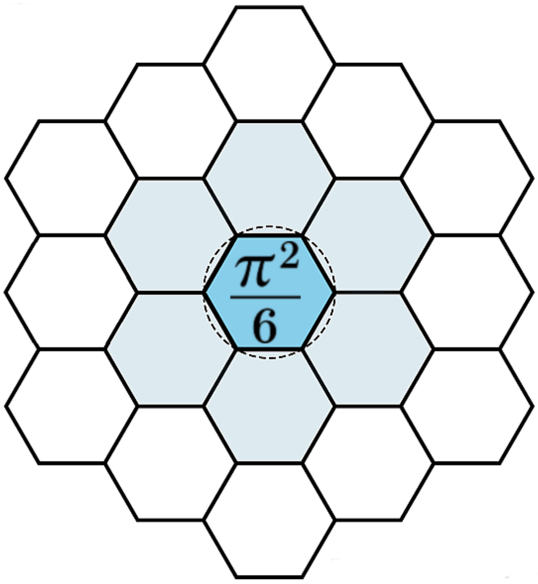

Abstract

This paper formulates Structural Regime Resolution (SRR) as a renormalization-group (RG) style coarse-graining flow acting on constraint fields defined over the Allen Orbital Lattice (AOL). SRR previously functioned as a description-control principle and was formalized as a coarse-graining operator mapping volumetric constraint transition zones into effective boundaries. Here we extend SRR into a scale-parameterized semigroup of maps, introduce block-lattice coarse-graining on the AOL, and define fixed points corresponding to stable effective laws. The resulting framework explains how sharp boundary conditions, quasiparticles, and jump-condition models arise as low-SRR RG images of higher-SRR volumetric mismatch resolution, and it provides a direct bridge from PFT microstructure to familiar effective theories.

Motivation

Papers I–VII established that:

- constraint transitions are volumetric at high SRR,
- boundaries are reduced images at low SRR,
- mismatch and PAL capacity define thickness and excitation,
- and SRR admits a coarse-graining operator mapping volumetric structure to effective boundary data.

The natural completion is to treat SRR not as a single operator, but as a *scale-indexed flow*:

$$\mathcal{R}_L : \mathcal{S} \to \mathcal{S}_{\text{eff}}(L)$$

that produces a family of effective descriptions at increasing resolution length L (or decreasing structural resolution).

This is structurally equivalent to the core role of renormalization: a mapping from microscopic rules to effective macroscopic laws.

AOL as a Graph Substrate

We treat the Allen Orbital Lattice as a discrete adjacency structure:

$$G = (V, E)$$

with nodes V and edges E. A physical or structural state is represented by fields defined on nodes and edges.

Definition 1 (AOL state space)**.** *Let $\mathcal{S}$ denote the set of admissible field configurations on G:*

$$\mathcal{S} = \{o(v), \phi(v), a(v), p(e), k(e), \dots\}_{v\in V,\, e\in E}$$

where o is occupancy, ϕ is phase, a is admissibility, p is PAL capacity, and additional fields may represent coupling, curvature class, or transport allowance.

SRR as a Coarse-Graining Semigroup

Definition 2 (SRR renormalization map)**.** *For each resolution length L (block size), define an SRR map:*

$$\mathcal{R}_L : \mathcal{S} \to \mathcal{S}_{\text{eff}}(L)$$

that replaces fine-grained fields on G by block-averaged or block-composed effective fields.

Proposition 1 (Semigroup property)**.** *If coarse-graining is performed in stages L_1 then L_2, the resulting map satisfies:*

$$\mathcal{R}_{L_2} \circ \mathcal{R}_{L_1} \approx \mathcal{R}_{L_2 L_1}$$

up to representation choices and normalization.

This expresses the core RG property: repeated SRR reduction composes into a single effective reduction.

Block-Lattice Construction on the AOL

Define a blocking procedure $\mathcal{B}_L$ that partitions nodes into blocks of diameter L:

$$\mathcal{B}_L : V \to \{B_i\}$$

with each block $B_i \subset V$.

For a field $f(v)$ defined on nodes, define an effective block field:

$$f_{\text{eff}}(B_i) = \mathcal{C}(\{f(v) : v \in B_i\})$$

where $\mathcal{C}$ is a chosen aggregation operator.

Remark 1. *The choice of $\mathcal{C}$ depends on the physical interpretation: averages for densities, circular means for phases, maxima for barrier-like constraints, and energy-minimizing compositions for compatibility fields.*

Coarse-Graining of Mismatch and PAL

Let $m(x)$ denote mismatch density and $p(x)$ PAL capacity density (Paper VII). Under coarse-graining to scale L, define:

$$m_{\text{eff}}(B_i) = \mathcal{C}(\{m(v) : v \in B_i\}), \qquad p_{\text{eff}}(B_i) = \mathcal{C}(\{p(v) : v \in B_i\}).$$

Definition 3 (Effective excitation density)**.** *Define effective excitation (dissipation) density:*

$$q_{\text{eff}}(B_i) = \max\left(0,\, m_{\text{eff}}(B_i) - p_{\text{eff}}(B_i)\right).$$

Proposition 2 (Preservation of mandatory dissipation)**.** *If excitation density is defined by $q = \max(0, m - p)$ at fine scale, then under monotone coarse-graining operators $\mathcal{C}$ the resulting effective excitation remains nonnegative and represents unresolved mismatch at scale L.*

Boundary Emergence as an RG Image

At high SRR, a transition zone is a finite set of nodes $\Omega \subset V$ where mismatch persists:

$$\Omega = \{v \in V : m(v) > 0\}$$

At low SRR (large L), the transition zone is represented by a boundary object b defined by jump data.

Definition 4 (Boundary extraction functional)**.** *Define a boundary extraction functional $\mathcal{B}_L$ acting on coarse-grained fields:*

$$\mathcal{B}_L(\mathcal{S}_{\text{eff}}(L)) = (x^*,\, \Delta\mathcal{C},\, Q_s)$$

where x^* is an effective boundary location, $\Delta\mathcal{C}$ are effective jump conditions, and Q_s is integrated excitation represented as a surface term.

Remark 2. *The location x^* may be defined as a maximizer of constraint gradient magnitude at scale L:*

$$x^* = \arg\max_x \|\nabla \mathcal{C}_{\text{eff}}(x)\|,$$

or the minimizer of an energy functional under two-dominion boundary constraints.

Fixed Points and Effective Laws

In RG theory, fixed points represent scale-invariant behavior. SRR admits analogous fixed points: stable effective descriptions that do not qualitatively change under additional coarse-graining.

Definition 5 (SRR fixed point)**.** *An effective state $\mathcal{S}^*$ is an SRR fixed point at scale L if:*

$$\mathcal{R}_L(\mathcal{S}^*) \approx \mathcal{S}^*$$

up to normalization or representation.

Proposition 3 (Boundary fixed points)**.** *Idealized jump-condition models (shocks, contact discontinuities, thin membranes, point particles) correspond to SRR fixed points when coarse-graining collapses internal structure into stable boundary parameters.*

Remark 3. *A fixed point does not imply the underlying reality is a boundary; it implies the boundary model is stable under further reduction at the chosen observational regime.*

Flow of Parameters Under SRR

Let θ denote a vector of effective parameters describing a reduced model:

$$\theta(L) = (\delta_{\text{eff}}(L),\, \Delta\mathcal{C}(L),\, Q_s(L),\, \dots)$$

Define the SRR flow:

$$\frac{d\theta}{d\ln L} = \beta_{\text{SRR}}(\theta)$$

where β_{SRR} is the SRR beta-functional describing how effective parameters change as resolution decreases.

Remark 4. *This form is not an import from external physics; it is the minimal calculus required to express "what changes when you describe the same transition at different SRR".*

Examples of SRR-RG Interpretation

Heliopause

High SRR: broad heated transition region with layered structure. Low SRR: effective heliopause boundary with jump conditions plus an integrated surface excitation term. SRR-RG predicts stability of the effective boundary only when the coarse-graining scale exceeds the internal layering scale.

Lightning

High SRR: stepped leader and streamer volumes. Low SRR: a thin line channel model with effective current and boundary-like discharge path. SRR-RG clarifies that "channel" is a reduced object.

Quantum criticality

High SRR: system-wide mismatch and divergent correlation length. Low SRR: phase diagram point or line. SRR-RG predicts that low-SRR collapse fails near criticality because the fixed point is scale-invariant in the wrong direction (critical fluctuations remain volumetric).

Quanta-scale barriers

High SRR: finite barrier region with admissibility decay. Low SRR: boundary conditions (infinite wall) as a stable reduction fixed point in certain approximations.

Canonical Statement

Remark 5 (SRR as AOL renormalization)**.** *Structural Regime Resolution defines a renormalization-group style coarse-graining flow on the Allen Orbital Lattice. Effective boundaries, jump conditions, and quasiparticle-like objects are stable low-SRR fixed points of this flow and are not ontological primitives.*

Conclusion

SRR is now formalized as:

- a scale-indexed family of coarse-graining maps on AOL fields,
- a semigroup composition property,
- a boundary extraction functional yielding effective discontinuity models,
- and a fixed-point language identifying stable effective laws.

This establishes the direct bridge from PFT microstructure (AOL, coherons, PAL) to the standard emergence of effective macroscopic theories, without reclassifying reduced descriptions as fundamental objects.

Glossary

AOL Allen Orbital Lattice, discrete substrate of PFT.

SRR Structural Regime Resolution, description-control parameter.

$\mathcal{R}_L$ SRR coarse-graining map at scale L.

$\mathcal{B}_L$ Blocking partition of AOL nodes into size-L blocks.

$m(x)$ Constraint mismatch density.

$p(x)$ PAL coherence capacity density.

$q(x)$ Excitation (dissipation) density.

Fixed point Effective description stable under further SRR reduction.

β_{SRR} Parameter flow functional under SRR.

Document Timestamp and Provenance

This document is part of Pattern Field Theory (PFT) and the Allen Orbital Lattice (AOL). It formulates Structural Regime Resolution as a renormalization group style coarse-graining flow on AOL-defined constraint fields, providing the next formal layer after SRR operators and AOL metrics.