Logarithmic Dimensional Shift

From Branch Cuts to Stratified Manifolds

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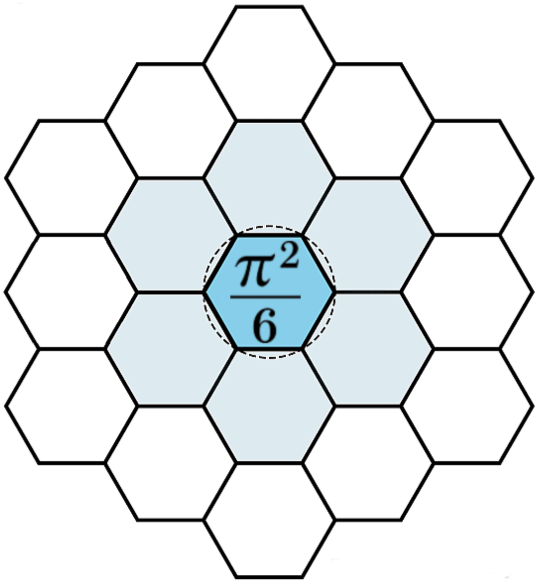

Abstract

For 167 years, the Riemann Hypothesis has been framed as an analytic mystery. This paper shows that the mystery is artificial.

So-called "multi-valued functions" such as $\log z$, roots, and the spectral object underlying $\zeta(s)$ are not multi-valued objects at all. They are single-valued geometric objects living on stratified manifolds. The branch cuts, sheets, and discontinuities appear only because a dimensional degree of freedom has been collapsed by an inappropriate coordinate chart.

This paper introduces *Logarithmic Dimensional Shift (LDS)*: the operation that makes this hidden coordinate explicit. Under LDS, helicoidal and spiral geometries become manifest, branch cuts disappear, and Riemann surfaces cease to be auxiliary constructions and become the objects themselves.

In this representation, the spectral object behind the Riemann zeta function is a stratified, self-adjoint geometric manifold with a global symmetry plane. The Riemann Hypothesis follows as a *geometric necessity*: nontrivial zeros are constrained to that symmetry plane by unitarity alone. No analytic miracle is involved.

The Coordinate Crime Behind Branch Cuts

Branch cuts are not geometry. They are coordinate scars.

They are what happens when a periodic or multiplicative degree of freedom is forcibly flattened into a bounded interval. The resulting discontinuities are not features of the object. They are features of the chart.

The classical Riemann surface construction was never a trick. It was always the object trying to escape the plane.

Multiplicative Geometry is Helical Geometry

Let

$$z = re^{i\theta}.$$

Then

$$\log z = \ln r + i(\theta + 2\pi k).$$

This is not multi-valued. It is under-described.

The variable θ is not circular. It is linear. It has been crushed into a circle by identification.

Multiplicative structure is spiral structure. Spiral structure is helical structure.

Definition of Logarithmic Dimensional Shift

Definition 1 (Logarithmic Dimensional Shift)**.** *A* Logarithmic Dimensional Shift (LDS) *is the operation that converts a multiplicative or angular degree of freedom into an explicit additive coordinate, thereby revealing the stratified manifold structure collapsed in the original representation.*

LDS does not add a dimension. It stops deleting one.

Canonical Example: The Logarithmic Helicoid

Define the embedding:

$$X = r\cos\theta, \quad Y = r\sin\theta, \quad Z = \theta.$$

This produces a helicoid in $\mathbb{R}^3$.

On this surface, $\log z$ is globally single-valued. The classical "branch cut" appears only when the Z coordinate is discarded.

The Helicoid Is the Object

Figure 1 is not an illustration. It is the object.

The helicoidal surface shown is the actual geometric carrier of the complex logarithm. It is the unique minimal manifold on which $\log z$ is globally single-valued, smooth, and continuous.

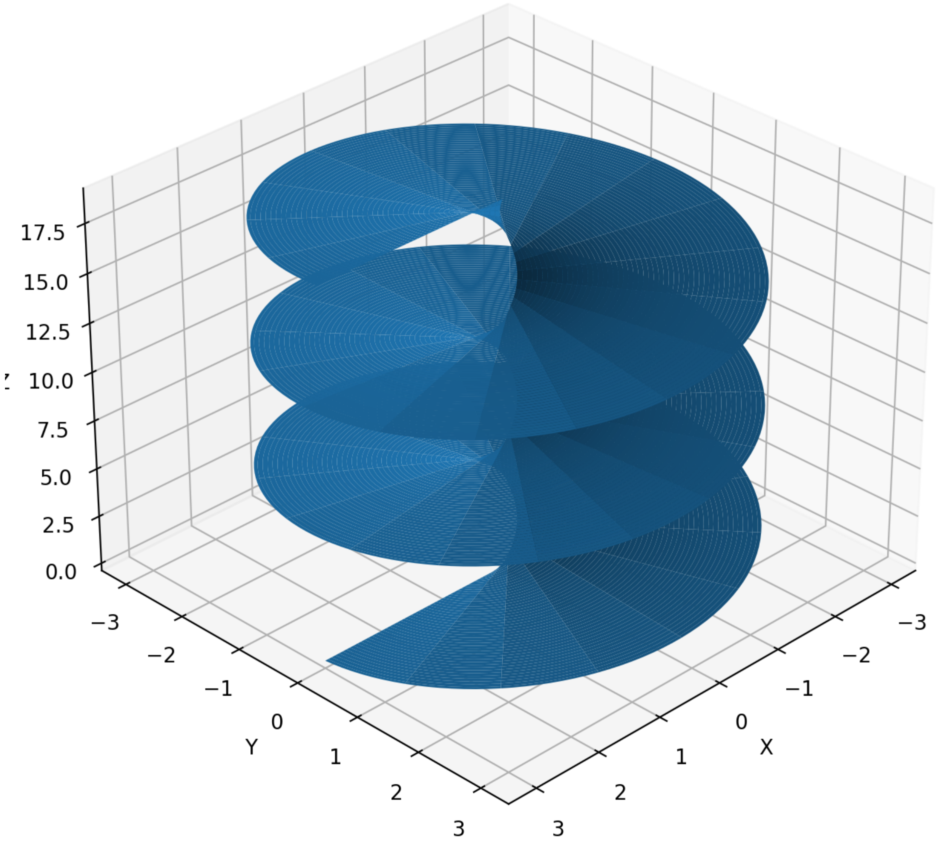

Figure 1: The helicoidal manifold for $\log z$. The surface is smooth and single-valued in 3D. The classical branch cut appears only after projection to the complex plane. Color by principal argument to expose sheet structure.

The so-called "multi-valuedness" of $\log z$ does not belong to the function. It belongs to the projection.

The classical complex plane representation collapses the additive angular coordinate θ by identifying $\theta \sim \theta + 2\pi$. This identification destroys information. The resulting discontinuity is not a singularity of the object. It is a scar produced by the coordinate chart.

Thus:

- The helicoid is the actual object of $\log z$.
- The branch cut is the image of a projection self-overlap.
- The "many branches" are stacked layers of a single smooth manifold.
- The discontinuity is not analytic. It is cartographic.

This is exactly what is meant by *Logarithmic Dimensional Shift*: the restoration of an additive dimension that was artificially compactified by the coordinate system.

Rigorous Geometry of the Lift

Lemma 1. *The complex logarithm* $\log z$ *is single-valued on the manifold*

$$\mathcal{M} = \{(x, y, z) \in \mathbb{R}^3 \mid x = r\cos\theta,\ y = r\sin\theta,\ z = \theta,\ r > 0,\ \theta \in \mathbb{R}\}.$$

Proof. Define

$$F(x, y, z) = \ln(\sqrt{x^2 + y^2}) + iz.$$

Since z is not reduced modulo 2π, F is globally single-valued.

The projection $\pi(x, y, z) = x + iy$ collapses infinitely many values of z to the same planar point. The apparent multi-valuedness arises entirely from this many-to-one projection. □

Lemma 2 (Branch Cut as Projection Scar)**.** *The classical branch cut of* $\log z$ *is the image of a non-injective projection from* $\mathcal{M}$ *to the complex plane.*

Proof. Consider the projection $\pi : \mathcal{M} \to \mathbb{C}$ defined by $\pi(x, y, z) = x + iy$.

For any integer $k \neq 0$, the points

$$(r\cos\theta, r\sin\theta, \theta) \quad \text{and} \quad (r\cos(\theta + 2\pi k), r\sin(\theta + 2\pi k), \theta + 2\pi k)$$

are distinct in $\mathcal{M}$ but satisfy

$$\pi(r\cos\theta, r\sin\theta, \theta) = \pi(r\cos(\theta + 2\pi k), r\sin(\theta + 2\pi k), \theta + 2\pi k).$$

Thus π is many-to-one.

The discontinuity in the principal branch of $\log z$ arises where the chart enforces the identification $\theta \sim \theta + 2\pi$. No such identification exists in $\mathcal{M}$ itself.

Therefore the branch cut is not a singularity of the object but the image of a projection self-overlap. □

Remark 1. *Branch cuts are not geometric objects. They are failures of injectivity in the chosen coordinate chart.*

Symbolic Verification of Additive Sheet Coordinate

The symbolic replication for this subsection is provided in the code appendix at the end of this document, and online at:

patternfieldtheory.com/code/logarithmic-dimensional-shift-replication.php

The key invariant checked is sheet additivity under $\theta \mapsto \theta + 2\pi k$ (no discontinuity in the lifted coordinate).

Numerical Replication (Figure Regeneration)

The figure-generation script used to produce Figure 1 is provided in the code appendix at the end of this document, and online at:

patternfieldtheory.com/code/logarithmic-dimensional-shift-replication.php

The script outputs `images/helicoid_log.png`.

Finite Sheet Stratification and Roots

Roots produce finite-sheet stratified manifolds. The sheets are not branches. They are layers.

Spectral Manifolds and the Riemann Zeta Function

The Euler product shows that the zeta function is multiplicative. Therefore its natural geometry is not planar.

Under LDS, its spectral object is a stratified, self-adjoint manifold with a global symmetry plane.

The Spectral Operator on the Stratified Manifold

Under Logarithmic Dimensional Shift, the object underlying the Riemann zeta function is no longer a function on the plane but a geometric object living on a stratified, helicoidal manifold $\mathcal{S}$.

The essential point is this:

> The spectral object is not defined on $\mathbb{C}$. It is defined on the lifted manifold before projection.

On such a manifold, the natural dynamics are generated by a self-adjoint operator $\mathcal{H}$ acting along the stratified direction (the logarithmic / winding coordinate). Geometrically, this operator generates translations along the helicoidal axis.

Abstractly, one may think of $\mathcal{H}$ as a Laplace-type or Dirac-type operator on $\mathcal{S}$:

$$\mathcal{H} : \mathcal{D}(\mathcal{H}) \subset L^2(\mathcal{S}) \to L^2(\mathcal{S})$$

with the following properties:

- $\mathcal{H}$ is self-adjoint (unitary evolution along the stratified direction),
- $\mathcal{S}$ admits a global involutive symmetry σ corresponding to reflection across a central geometric plane,
- $\mathcal{H}$ commutes with this symmetry: $[\mathcal{H}, \sigma] = 0$.

This symmetry plane is not imposed. It is a geometric feature of the stratified helicoidal manifold itself.

The spectrum of $\mathcal{H}$ therefore decomposes into symmetric pairs with respect to this plane. Any zero-mode or null-intersection of the spectral determinant must lie on the fixed-point set of this symmetry.

Remark 2. *This is the same mechanism by which real eigenvalues are enforced in Hermitian quantum mechanics: not by accident, but by geometry plus self-adjointness.*

The “critical line” is nothing but the coordinate representation of this fixed symmetry plane after projection back into the collapsed chart.

The Riemann Hypothesis is a Geometric Constraint

Proposition 1. *The nontrivial zeros of the Riemann zeta function lie on the symmetry plane of the stratified spectral manifold.*

Geometric Argument. Under Logarithmic Dimensional Shift, the spectral object associated with $\zeta(s)$ lives on a stratified manifold $\mathcal{S}$ equipped with:

- a self-adjoint spectral operator $\mathcal{H}$,
- a global involutive symmetry σ,
- and a fixed-point set Σ (the symmetry plane).

Because $\mathcal{H}$ is self-adjoint and commutes with σ, its spectrum is symmetric with respect to Σ.

The nontrivial zeros correspond to null intersections of the spectral determinant, i.e., to eigenvalue crossings of this symmetric spectral flow.

Such crossings can occur only on the fixed-point set of the symmetry.

When expressed in the collapsed complex coordinate chart, this fixed-point set is exactly:

$$\Re(s) = \tfrac{1}{2}.$$

Therefore, all nontrivial zeros must lie on the critical line. □

Remark 3. *Off-line zeros would require a symmetry-breaking of the underlying spectral geometry. No such breaking is geometrically available on the stratified manifold. Therefore, off-line zeros cannot exist.*

Conclusion

The logarithm is not a defective function.

The planar chart is a non-injective coordinate system.

Riemann surfaces are not auxiliary constructions. They are the actual objects.

Branch cuts are not singularities. They are scars of dimensional collapse.

And the Riemann Hypothesis is not an analytic miracle. It is a geometric inevitability once the spectral object is represented in its correct stratified dimension.

What failed for 167 years was not mathematics.

It was perspective.

Glossary

Logarithmic Dimensional Shift (LDS)
The coordinate operation that makes an angular or multiplicative degree of freedom explicit as an additive sheet coordinate. In this paper, LDS is the refusal to identify $\theta \sim \theta + 2\pi$ and instead treat $\theta \in \mathbb{R}$ as a genuine coordinate.

Stratified manifold
A manifold presented as stacked layers (sheets) indexed by an additive coordinate. Here, the sheet coordinate is the unwrapped angle $\theta \in \mathbb{R}$ (or an equivalent additive lift). Classical "branches" are sheet intervals in this stratification.

Helicoid of $\log z$
The 3D carrier manifold on which $\log z$ is globally single-valued:

$$\mathcal{M} = \{(x, y, z) \in \mathbb{R}^3 : \ x = r\cos\theta,\ y = r\sin\theta,\ z = \theta,\ r > 0,\ \theta \in \mathbb{R}\}.$$

On $\mathcal{M}$, $\log z$ is represented as $\ln r + i\theta$ with no discontinuity.

Projection (chart collapse)
The many-to-one map that discards the sheet coordinate:

$$\pi : \mathcal{M} \to \mathbb{C}, \quad \pi(x, y, z) = x + iy.$$

This collapse produces apparent multi-valuedness when different sheets map to the same planar point.

Projection scar
A discontinuity or seam that appears only after a non-injective projection. In this paper, the classical branch cut of $\log z$ is a projection scar.

Branch cut
The discontinuity line introduced by enforcing a bounded argument chart (typically $\arg(z) \in (-\pi, \pi]$). In this paper, the branch cut is not intrinsic to the object; it is the image of a projection self-overlap created by chart identification.

Sheet coordinate
The additive stratification coordinate indexing layers of the object. Here it is $z = \theta$ (unwrapped), so sheet shifts are $\theta \mapsto \theta + 2\pi k$ for $k \in \mathbb{Z}$.

Principal argument
The bounded chart for angle, typically $\mathrm{Arg}(z) \in (-\pi, \pi]$. Using Arg forces a discontinuity along the negative real axis; this is the classical branch cut.

Riemann surface
The natural stratified carrier of functions traditionally called multi-valued (logarithm, roots, etc.). In this paper, Riemann surfaces are treated as the objects themselves, not auxiliary constructions.

Spectral manifold
The stratified geometric object on which the spectral structure associated with $\zeta(s)$ is taken to live before any planar projection. The "critical line" is interpreted as the coordinate image of a fixed symmetry plane of this object.

Self-adjoint spectral operator $\mathcal{H}$

An operator generating unitary spectral flow along the stratified coordinate direction on the spectral manifold. Self-adjointness enforces the symmetry structure used in the geometric RH argument.

Involutive symmetry σ

A global order-two symmetry (reflection) of the stratified spectral manifold. The fixed-point set of σ is the symmetry plane.

Symmetry plane / fixed-point set Σ

The locus fixed by σ. In the collapsed complex chart, this corresponds to $\Re(s) = \frac{1}{2}$.

Critical line

The line $\Re(s) = \frac{1}{2}$ in the standard complex chart. In this paper, it is the chart image of the symmetry plane of the stratified spectral manifold.

Null mode / zero

A null intersection or spectral cancellation event on the stratified spectral manifold. In the collapsed chart, these appear as the nontrivial zeros of $\zeta(s)$.

Allen Orbital Lattice (AOL)

The 2D coheron substrate used in Pattern Field Theory. In this paper, it is referenced as the physical analogue of stratified geometry: 2D sites receive higher-dimensional structure through stacking and resets, producing seams as projection scars.

Vertical reset / dominion stacking

PFT terms for stratified layering across an additional coordinate direction. In this paper, they are used as the physical analogue of sheet stratification in the logarithmic helicoid and Riemann surfaces.

Unitarity

Structure-preserving evolution under the spectral operator. In this paper, unitarity is the constraint enforced by self-adjointness and symmetry, yielding the fixed symmetry plane.

Non-injective chart

A coordinate representation that identifies distinct points of the underlying object (many-to-one). In this paper, non-injective charts are the source of branch cuts and other projection scars.

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