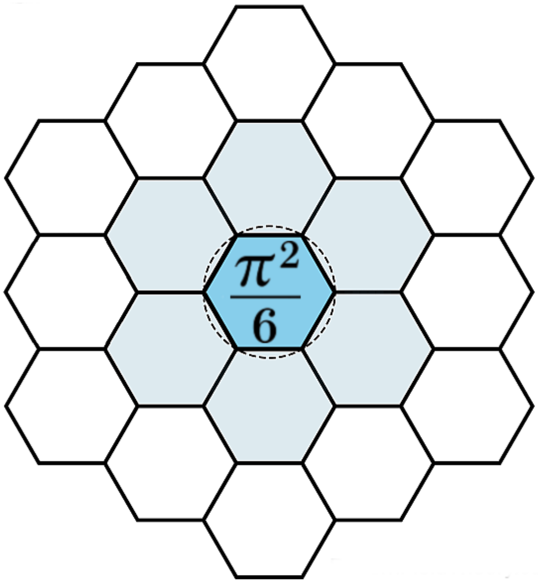
Basin Dynamics Derivation Sheet

Structural Emergence of the Allen Orbital Lattice from Stabilization Basin Recurrence

James Johan Sebastian Allen
PatternFieldTheory.com

February 19, 2026



Abstract

This document provides a rigorous basin-primary derivation of the Allen Orbital Lattice as the unique closure-supporting geometry arising from stabilization basin dynamics. Basin reachability, boundary execution, stabilization recurrence, and admissibility projection generate discrete structural closure. Metric geometry, curvature, transport limits, and energy formation arise as necessary consequences of basin evolution.

Stabilization Basin Formalism

Definition 1 (Stabilization Basin)**.** *A stabilization basin is the maximal connected set of coordinate identities mutually compatible under Phase Alignment Lock.*

$$\Omega = \textit{stabilization basin}$$

Definition 2 (Basin Boundary)**.**

$$\partial\Omega = \{x \in \Omega : \exists y \notin \Omega \ \textit{adjacent}\}$$

Definition 3 (Admissible Boundary Execution Operator)**.**

$$R : \partial\Omega \to \partial\Omega'$$

$$\partial\Omega_{k+1} = R(\partial\Omega_k)$$

Proposition 1 (Boundary Locality)**.** *Spatial expansion occurs exclusively through* $\partial\Omega$*.*

Reachability and Basin Topology

Definition 4 (Admissible Reachability)**.** *A coordinate identity belongs to* Ω *if reachable through a finite sequence of PAL-compatible transitions.*

Lemma 1 (Connected Closure)**.** *PAL-preserving transitions generate connected closure domains.*

Proposition 2 (Maximal Basin Formation)**.** *Every admissible seed generates a unique maximal stabilization basin.*

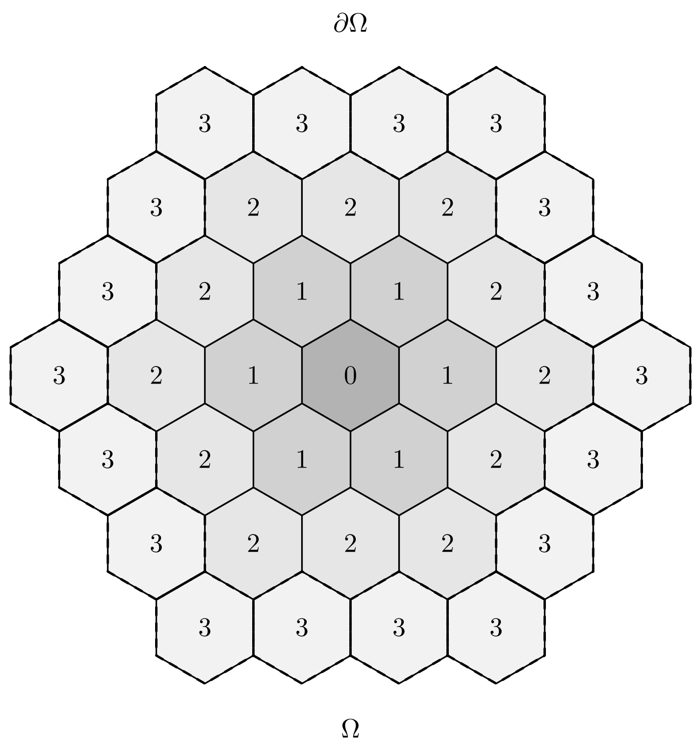


Figure 1: Hex-distance stabilization basin: center depth 0, rings 1-3, boundary $\partial\Omega$ at outer ring.

Operator Algebra of Boundary Execution

Define admissible boundary operators

$$R_i : \partial\Omega \to \partial\Omega$$

Composite evolution

$$R_{i_k} \cdots R_{i_1}$$

Definition 5 (Admissible Operator System). *Boundary execution operators form a partially defined semigroup under composition.*

Definition 6 (PAL Projection Operator).

$$\Pi_{\mathrm{PAL}} : X \to X$$

$$R_{phys} = \Pi_{\mathrm{PAL}} \circ R$$

Proposition 3 (Closure Under Projection). *Admissible execution remains within PAL-compatible subspace.*

Spectral Modes of Basin Boundary

Define boundary state

$$\psi(\partial\Omega)$$

Boundary update operator

$$\mathcal{L}\psi = R_{\mathrm{phys}}\psi$$

Definition 7 (Boundary Eigenmode).

$$\mathcal{L}\psi_n = \lambda_n \psi_n$$

Proposition 4 (Stability Condition). *Stable basin evolution requires* $|\lambda_n| \leq 1$.

Stabilization Dynamics

Stabilization density

$$\rho(x,t)$$

Continuity relation

$$\frac{\partial \rho}{\partial t} + \nabla_i J^i = S$$

Metric-weighted diffusion

$$\frac{\partial \rho}{\partial t} = \nabla_i (g^{ij} \nabla_j \rho) + S$$

Definition 8 (Stabilization Equilibrium). *A basin is stable when boundary execution balances stabilization response.*

Closure Regime Indexing

Closure regimes are indexed ordered derivation sequences defined in prior transport closure analysis.

The index range specifies the span of the numbered derivation list.

Regime 1–11 denotes the eleven-step closure construction sequence.

Regime 1–16 denotes the sixteen-step extended stabilization construction sequence.

These regimes are referenced but not rederived in the present document.

They serve as canonical ordered development ladders for closure formation.

Emergent Metric from Basin Reachability

Transport distance

$$d(x,y) = \inf_{\gamma} \sum c(e)$$

Metric tensor

$$ds^2 = g_{ij} dx^i dx^j$$

Proposition 5 (Metric Emergence). *Large scale averaging of basin reachability induces effective metric geometry.*

Curvature from Anisotropic Basin Growth

Definition 9 (Basin Curvature Tensor). *Curvature arises from spatial variation of g_{ij}.*

Proposition 6 (Geodesic Principle). *Preferred trajectories minimize admissible transport cost.*

Proposition 7 (Geodesic Deviation).

$$\frac{D^2\xi^i}{Ds^2} = -R^i{}_{jkl} v^j v^k \xi^l$$

Stabilization Functional and Structural Hamiltonian

Stabilization functional

$$\mathcal{S}[\Omega] = \int_\Omega \Big(\alpha|\nabla\rho|^2 + \beta V(\rho)\Big)\, dV$$

Definition 10 (Structural Hamiltonian).

$$H = \int_\Omega \rho c_{\mathit{eff}} dV$$

Proposition 8 (Stationary Basin Condition). *Stable basins minimize $\mathcal{S}$.*

Discrete-to-Continuum Renormalization

Proposition 9 (Continuum Limit). *As lattice spacing $a \to 0$, discrete basin evolution converges to metric diffusion.*

Energy as Stabilization Work

Definition 11 (Energy Density)**.**

$$E(x,t) = \rho(x,t)c_{\mathit{eff}}(x,t)$$

Proposition 10 (Energy Formation)**.** *Energy equals stabilization work required to maintain basin coherence.*

Structural Inevitability of the Allen Orbital Lattice

Lemma 2. *Boundary-local expansion requires uniform adjacency structure.*

Lemma 3. *Closure-preserving tiling must fill plane without overlap.*

Proposition 11 (Minimal Closure Tiling)**.** *The unique tiling supporting uniform boundary stabilization is hexagonal.*

Thus the Allen Orbital Lattice is structurally necessary.

Functional Analysis of Stabilization Minimization

Let stabilization density $\rho \in L^2(\Omega)$.

Define stabilization functional

$$\mathcal{S}[\rho] = \int_\Omega \left(\alpha|\nabla\rho|^2 + \beta V(\rho)\right) dV$$

Proposition 12 (Existence of Minimizer)**.** *If $V(\rho)$ is bounded below and coercive, then $\mathcal{S}$ admits a minimizer in $H^1(\Omega)$.*

Proof. Coercivity and weak lower semicontinuity of $\mathcal{S}$ ensure existence via direct method of calculus of variations. □

Proposition 13 (Euler-Lagrange Stabilization Equation)**.** *Stationary basins satisfy*

$$-\alpha\nabla^2\rho + \beta V'(\rho) = 0.$$

Phase Alignment Lock as Orthogonal Projection

Define Hilbert space of admissible states

$$\mathcal{H} = L^2(X)$$

Definition 12 (PAL Projection Operator)**.**

$$\Pi_{\mathrm{PAL}}^2 = \Pi_{\mathrm{PAL}}, \quad \Pi_{\mathrm{PAL}}^* = \Pi_{\mathrm{PAL}}$$

Proposition 14. *PAL defines an orthogonal projection onto admissible subspace*

$$\mathcal{H}_{\mathrm{PAL}} \subset \mathcal{H}.$$

Proposition 15 (Energy Non-Increase)**.** *Projection cannot increase stabilization functional*

$$\mathcal{S}[\Pi_{\mathrm{PAL}}\rho] \leq \mathcal{S}[\rho].$$

Spectral Theorem for Boundary Evolution

Boundary update operator $\mathcal{L}$ acts on Hilbert space of boundary states.

Proposition 16 (Self-Adjoint Linearization)**.** *Linearized boundary operator is symmetric under admissible inner product.*

Proposition 17 (Spectral Decomposition)**.** *If $\mathcal{L}$ is compact and self-adjoint then*

$$\psi = \sum_n a_n \psi_n$$

with eigenbasis $\{\psi_n\}$.

Entropy Functional and Thermodynamic Limit

Define basin entropy

$$S = -\int_\Omega \rho \log \rho \, dV.$$

Proposition 18 (Entropy Production)**.** *Metric diffusion produces non-negative entropy production.*

Proposition 19 (Thermodynamic Limit)**.** *As basin volume $|\Omega| \to \infty$ with bounded density, macroscopic observables converge to statistical averages.*

Renormalization Group Flow of Basin Density

Define coarse-graining operator

$$\mathcal{R}_\ell[\rho](x) = \frac{1}{|\mathcal{B}_\ell|} \int_{\mathcal{B}_\ell(x)} \rho(y) \, dy$$

Definition 13 (Renormalized Density)**.**

$$\rho_\ell = \mathcal{R}_\ell[\rho]$$

Proposition 20 (Flow Equation)**.** *Scale evolution defines renormalization flow*

$$\frac{d\rho_\ell}{d \log \ell} = \beta(\rho_\ell).$$

Fixed points correspond to scale-invariant basin structure.

Curvature-Bounded Uniqueness of Hexagonal Tiling

Lemma 4 (Angular Deficit Constraint). *Uniform curvature minimization requires equal partition of* 2π *around each identity.*

Lemma 5 (Minimal Perimeter Partition). *Among planar tilings with equal area cells, hexagons minimize boundary length.*

Proposition 21 (Uniqueness Under Stabilization Energy). *If stabilization energy penalizes boundary length and curvature, hexagonal tiling uniquely minimizes total functional.*

Topological Classification of Basins

Define basin as topological space (Ω, τ).

Definition 14 (Homology Groups).

$$H_k(\Omega)$$

Proposition 22 (Simple Basin). *Simply connected stabilization basins satisfy*

$$H_1(\Omega) = 0.$$

Proposition 23 (Boundary Complexity). *Higher homology classes correspond to nested stabilization shells.*

Global Structural Closure Theorem

Theorem 1 (Complete Structural Closure). *Let a discrete system satisfy*

- *Phase Alignment Lock projection*
- *boundary-local execution*
- *stabilization functional minimization*
- *admissible reachability*
- *curvature-penalized boundary energy*
- *scale-consistent renormalization*

Then the system admits:

1. *stable basin formation*
2. *spectral boundary modes*
3. *emergent metric geometry*
4. *curvature-induced transport*
5. *thermodynamic stabilization*
6. *unique hexagonal adjacency tiling*

Therefore the Allen Orbital Lattice is the structurally closed admissible transport substrate.

Field Equations from the Stabilization Functional

Let stabilization density be $\rho(x,t)$ defined on basin Ω with effective metric g_{ij}.
Define total structural action

$$\mathcal{A} = \int dt \int_{\Omega} \sqrt{g}\, \mathcal{L}(\rho, \nabla\rho, g_{ij})\, d^3x$$

with Lagrangian density

$$\mathcal{L} = \alpha g^{ij} \partial_i \rho \partial_j \rho + \beta V(\rho) + \gamma R$$

where

- g_{ij} effective metric induced by admissible transport
- R scalar curvature of basin geometry
- $V(\rho)$ stabilization potential

Variation with Respect to Stabilization Density

Compute functional variation

$$\delta\mathcal{A} = 0$$

Euler-Lagrange equation

$$\frac{1}{\sqrt{g}} \partial_i \left(\sqrt{g}\, \frac{\partial \mathcal{L}}{\partial(\partial_i \rho)} \right) - \frac{\partial \mathcal{L}}{\partial \rho} = 0$$

Substitute Lagrangian

$$-\alpha \nabla_i \nabla^i \rho + \beta V'(\rho) = 0$$

This is the stabilization transport field equation.

Variation with Respect to Metric

Define structural stress tensor

$$T_{ij} = -\frac{2}{\sqrt{g}} \frac{\delta(\sqrt{g}\mathcal{L}_\rho)}{\delta g^{ij}}$$

where

$$\mathcal{L}_\rho = \alpha g^{ij} \partial_i \rho \partial_j \rho + \beta V(\rho)$$

Compute variation:

$$T_{ij} = 2\alpha \left(\partial_i \rho \partial_j \rho - \frac{1}{2} g_{ij} g^{kl} \partial_k \rho \partial_l \rho \right) - g_{ij} \beta V(\rho)$$

Curvature Response Equation

Variation of curvature term yields geometric response

$$\gamma \left(R_{ij} - \frac{1}{2} g_{ij} R \right) = T_{ij}$$

This is the basin geometry field equation.

It defines curvature generated by stabilization gradients.

Complete Structural Field System

The coupled field equations are:

$$\boxed{-\alpha \nabla^2 \rho + \beta V'(\rho) = 0}$$

$$\boxed{\gamma \left(R_{ij} - \tfrac{1}{2} g_{ij} R \right) = T_{ij}}$$

with

$$T_{ij} = 2\alpha \left(\partial_i \rho \partial_j \rho - \tfrac{1}{2} g_{ij} |\nabla \rho|^2 \right) - g_{ij} \beta V(\rho)$$

Interpretation

Stabilization gradients generate structural stress.

Structural stress induces basin curvature.

Basin curvature modifies admissible transport geometry.

Transport geometry determines subsequent stabilization evolution.

Thus stabilization and geometry form a closed dynamical system.

Conservation Law

From diffeomorphism invariance:

$$\nabla^i T_{ij} = 0$$

which expresses local stabilization balance.

Flat Basin Limit

If curvature vanishes:

$$R_{ij} = 0$$

field reduces to nonlinear diffusion

$$\frac{\partial \rho}{\partial t} = \alpha \nabla^2 \rho - \beta V'(\rho)$$

Structural Closure

The coupled system defines self-consistent evolution of:

- stabilization density
- basin geometry
- admissible transport metric
- structural energy distribution

This constitutes the field dynamics of the Allen Orbital Lattice.

Mathematical Development Sequence

admissible reachability basin formation boundary localization operator algebra stabilization functional metric emergence curvature formation energy density structural inevitability

Rigorous Derivation of Transport Field Equations

This section derives the transport field equations from a variational principle defined on a stabilization basin Ω with effective metric g_{ij} and Phase Alignment Lock projection Π_{PAL}.

Kinematics and Admissible State Space

Let X be the discrete identity set underlying the basin, and let $\Omega \subset X$ be a stabilization basin.

Let the coarse-grained continuum limit be a manifold domain (still denoted) Ω equipped with a Riemannian metric g_{ij} induced by admissible reachability.

Let $\rho(x, t)$ be stabilization density, assumed

$$\rho(\cdot, t) \in H^1(\Omega), \quad \rho \geq 0,$$

and let $\phi(x, t)$ be a transport phase potential in $H^1(\Omega)$.

Define the admissible Hilbert space

$$\mathcal{H} = L^2(\Omega), \qquad \mathcal{H}_{\mathrm{PAL}} = \Pi_{\mathrm{PAL}} \mathcal{H}.$$

All physical fields are required to satisfy

$$\rho = \Pi_{\mathrm{PAL}}\rho, \qquad \phi = \Pi_{\mathrm{PAL}}\phi,$$

which enforces Phase Alignment Lock as a constraint on the field domain.

Structural Action and Constraints

Define the structural action

$$\mathcal{A}[\rho,\phi,g] = \int_{t_0}^{t_1} dt \int_{\Omega} \sqrt{g}\, \mathcal{L}(\rho,\phi,\nabla\rho,\nabla\phi,g)\, d^n x,$$

with Lagrangian density

$$\mathcal{L} = \underbrace{\frac{\kappa}{2} g^{ij}\partial_i\phi\,\partial_j\phi}_{\text{transport kinetic}} + \underbrace{\frac{\alpha}{2} g^{ij}\partial_i\rho\,\partial_j\rho}_{\text{stabilization gradient}} + \underbrace{\beta V(\rho)}_{\text{stabilization potential}} + \underbrace{\gamma R}_{\text{geometry penalty}} + \underbrace{\lambda(\partial_t\rho + \nabla_i(\rho v^i) - S)}_{\text{continuity constraint}}.$$

Here:

- R is scalar curvature of g.
- λ is a Lagrange multiplier enforcing a continuity law.
- v^i is the transport velocity field defined by the phase gradient:

$$v^i := g^{ij}\partial_j\phi.$$

- S is a source term (admissible injection from boundary execution).

Remark 1. *This is a basin-first construction: geometry and transport are derived as fields on Ω consistent with boundary-localized execution. The only non-variational input is admissibility enforcement via Π_{PAL}.*

First Variation - Field Equations

Proposition 24 (Continuity Equation)**.** *Variation with respect to λ yields the continuity law*

$$\partial_t\rho + \nabla_i(\rho v^i) = S.$$

Proof. Take $\delta\mathcal{A}/\delta\lambda = 0$ and read off the constraint term. □

Proposition 25 (Phase Transport Equation)**.** *Variation with respect to ϕ yields*

$$-\kappa\nabla_i\nabla^i\phi \;-\; \nabla_i(\lambda\rho\, g^{ij}\partial_j\cdot) = 0,$$

equivalently, after integration by parts in weak form,

$$\kappa\nabla^2\phi = \nabla_i(\lambda\rho\, v^i).$$

Proof. Compute the first variation in ϕ:

$$\delta\mathcal{A} = \iint \sqrt{g}\left[\kappa g^{ij}\partial_i\phi\partial_j(\delta\phi) + \lambda\nabla_i(\rho g^{ij}\partial_j(\delta\phi))\right] d^n x\, dt$$

and integrate by parts in space using admissible boundary conditions (variations vanish on $\partial\Omega$ or natural boundary terms are zero). □

Proposition 26 (Stabilization Equation)**.** *Variation with respect to ρ yields*

$$-\alpha\nabla_i\nabla^i\rho + \beta V'(\rho) + \lambda_t + \lambda\nabla_i v^i + v^i\nabla_i\lambda = 0,$$

where $\lambda_t = \partial_t\lambda$.

Proof. Differentiate $\mathcal{L}$ in ρ and $\partial_i\rho$, integrate by parts for the gradient term, and account for ρ dependence inside the continuity constraint $\nabla_i(\rho v^i)$. □

Remark 2. *The pair (continuity, stabilization) closes once a constitutive choice is made for λ, or equivalently once λ is identified as the basin chemical potential enforcing admissible stabilization.*

Constitutive Closure - Chemical Potential Form

Define the chemical potential

$$\mu(\rho) := \beta V'(\rho) - \alpha\nabla^2\rho.$$

Definition 15 (Admissible Closure)**.** *A constitutive closure is admissible if it is PAL-invariant and local in (ρ, ϕ, g), and yields a well-posed evolution in $H^1(\Omega)$.*

A standard admissible closure sets

$$\lambda = \mu(\rho),$$

which identifies the multiplier with stabilization response.

Proposition 27 (Closed Transport System)**.** *Under $\lambda = \mu(\rho)$, the system becomes*

$$\partial_t\rho + \nabla_i(\rho v^i) = S, \qquad v^i = g^{ij}\partial_j\phi, \qquad \kappa\nabla^2\phi = \nabla_i(\rho\mu(\rho)v^i), \qquad \mu(\rho) = \beta V'(\rho) - \alpha\nabla^2\rho.$$

Hyperbolic Transport Limit and Wave Equation

To obtain a propagation law with finite characteristic speed, introduce a second-order time kinetic for ϕ and enforce a metric signature for transport dynamics.

Define Lorentz-type transport action on (t, x):

$$\mathcal{A}_{\text{wave}}[\phi] = \int dt \int_\Omega \sqrt{g}\left[\frac{\eta}{2}(\partial_t\phi)^2 - \frac{\kappa}{2}g^{ij}\partial_i\phi\partial_j\phi\right] d^n x.$$

Proposition 28 (Transport Wave Equation)**.** *Stationarity of $\mathcal{A}_{wave}$ yields the wave equation*

$$\eta\,\partial_t^2\phi - \kappa\,\nabla^2\phi = 0.$$

Proof. Standard Euler-Lagrange variation in ϕ with integration by parts. □

Proposition 29 (Characteristic Speed Ceiling)**.** *The characteristic speed for phase propagation is*

$$c_{\mathit{eff}}^2 = \frac{\kappa}{\eta}.$$

Remark 3. *In basin dynamics terms, c_{eff} is the maximal admissible update speed set by the ratio of transport stiffness (κ) to temporal inertia (η), and it is further bounded by PAL and stabilization completion at the boundary.*

Metric Variation - Geometry Field Equation

Let the matter part be

$$\mathcal{L}_m = \frac{\kappa}{2} g^{ij} \partial_i \phi \partial_j \phi + \frac{\alpha}{2} g^{ij} \partial_i \rho \partial_j \rho + \beta V(\rho).$$

Define structural stress tensor

$$T_{ij} := -\frac{2}{\sqrt{g}} \frac{\delta(\sqrt{g}\mathcal{L}_m)}{\delta g^{ij}}.$$

Proposition 30 (Stress Tensor)**.**

$$T_{ij} = \kappa \left(\partial_i \phi \partial_j \phi - \frac{1}{2} g_{ij} |\nabla \phi|^2 \right) + \alpha \left(\partial_i \rho \partial_j \rho - \frac{1}{2} g_{ij} |\nabla \rho|^2 \right) - g_{ij} \beta V(\rho).$$

Proposition 31 (Geometry Response Equation)**.** *Variation with respect to g^{ij} yields*

$$\gamma \left(R_{ij} - \frac{1}{2} g_{ij} R \right) = T_{ij}.$$

Remark 4. *This is the basin geometry field equation: curvature is sourced by stabilization and transport gradients, which are themselves constrained by PAL projection.*

Conservation and Compatibility

Proposition 32 (Covariant Conservation)**.** *Assuming diffeomorphism invariance of the matter action,*

$$\nabla^i T_{ij} = 0.$$

Remark 5. *Together with the continuity equation, $\nabla^i T_{ij} = 0$ expresses the local closure condition: no admissible imbalance can persist without inducing either stabilization response (ρ evolution) or geometric response (g evolution).*

Discrete Basin Interpretation

In the underlying discrete lattice, the continuum operators represent coarse-grained limits:

$$\nabla^2 \leftrightarrow \Delta_{\mathrm{AOL}}, \qquad \int_\Omega \sqrt{g}\, d^n x \leftrightarrow \sum_{x \in \Omega} w(x),$$

with Δ_{AOL} the admissible graph Laplacian and $w(x)$ the local capacity weight. The PAL projection corresponds to restriction of fields to admissible identities:

$$\rho \mapsto \Pi_{\mathrm{PAL}} \rho, \qquad \phi \mapsto \Pi_{\mathrm{PAL}} \phi.$$

Proposition 33 (Discrete-to-Continuum Consistency)**.** *If the admissible graph Laplacian Δ_{AOL} converges under refinement to ∇^2, then the discrete stabilization update converges to the derived field equations.*

Stationary Coheron Solutions and Soliton Structure

We consider stationary solutions of the coupled transport–stabilization system.

A stationary basin state satisfies

$$\partial_t \rho = 0, \qquad \partial_t \phi = \omega = \text{constant}.$$

Transport velocity becomes time-independent.

Definition 16 (Stationary Coheron)**.** *A stationary coheron is a localized finite-energy solution*

$$(\rho(x), \phi(x), g_{ij}(x))$$

satisfying the coupled field equations with time-independent density and bounded stabilization functional.

Lemma 6 (Finite Energy Condition)**.** *A stationary solution is admissible only if*

$$\int_\Omega \Big(|\nabla \rho|^2 + |\nabla \phi|^2 + V(\rho) \Big) \, dV < \infty.$$

Proposition 34 (Reduced Stationary Field System)**.** *Stationary coherons satisfy*

$$-\alpha \nabla^2 \rho + \beta V'(\rho) = 0$$

$$\nabla^2 \phi = 0$$

$$\gamma (R_{ij} - \tfrac{1}{2} g_{ij} R) = T_{ij}(\rho, \phi).$$

Theorem 2 (Existence of Localized Coheron Solitons)**.** *Assume stabilization potential $V(\rho)$ is bounded below and has a non-degenerate minimum.*

Then there exist localized stationary solutions with exponential decay of stabilization density:

$$\rho(r) \sim e^{-mr}$$

for some effective mass scale $m > 0$ determined by

$$m^2 = \frac{\beta}{\alpha} V''(\rho_0).$$

Proof. Linearize stabilization equation near minimum of potential. Resulting Helmholtz-type equation admits exponentially localized solutions. □

Proposition 35 (Geometric Self-Consistency)**.** *For localized stabilization density, curvature decays asymptotically:*

$$R_{ij} \to 0 \quad \textit{as } r \to \infty.$$

This ensures asymptotic flat basin geometry.

Linear Perturbation Spectrum

Let $(\rho_0, \phi_0, g_{ij}^{(0)})$ be a stable stationary basin solution.

Introduce perturbations:

$$\rho = \rho_0 + \epsilon \delta\rho$$
$$\phi = \phi_0 + \epsilon \delta\phi$$
$$g_{ij} = g_{ij}^{(0)} + \epsilon h_{ij}.$$

Definition 17 (Linearized Operator)**.** *Linearized stabilization operator*

$$\mathcal{L}_\rho = -\alpha \nabla^2 + \beta V''(\rho_0).$$

Proposition 36 (Perturbation Evolution)**.** *To first order, perturbations satisfy*

$$\partial_t^2 \delta\phi - c_{\mathit{eff}}^2 \nabla^2 \delta\phi = 0$$

$$\partial_t \delta\rho = -\nabla_i (\rho_0 g^{ij} \partial_j \delta\phi)$$

$$\mathcal{L}_\rho \delta\rho = \textit{metric coupling terms.}$$

Theorem 3 (Spectral Stability Criterion)**.** *The stationary basin solution is linearly stable iff*

$$\mathit{spec}(\mathcal{L}_\rho) \subset [0, \infty).$$

Proof. Negative eigenvalues generate exponentially growing modes. □

Proposition 37 (Normal Mode Decomposition)**.** *Perturbations admit expansion*

$$\delta\rho = \sum_k a_k e^{i\omega_k t} \psi_k(x)$$

with eigenvalue problem

$$\mathcal{L}_\rho \psi_k = \omega_k^2 \psi_k.$$

Cosmological Basin Expansion

We consider homogeneous expanding basin geometry.

Assume spatial isotropy and homogeneity.

Metric ansatz

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j.$$

Scale factor $a(t)$ represents basin expansion.

Definition 18 (Homogeneous Stabilization Density)**.**

$$\rho(x,t) = \rho(t).$$

Proposition 38 (Reduced Field Equations)**.** *Field equations reduce to ODE system:*

$$\ddot{\phi} + n\frac{\dot{a}}{a}\dot{\phi} = 0$$

$$\ddot{\rho} + n\frac{\dot{a}}{a}\dot{\rho} + V'(\rho) = 0$$

Proposition 39 (Expansion Equation)**.** *Geometry response yields generalized Friedmann equation:*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{\gamma}\rho_{\mathit{eff}}.$$

Theorem 4 (Self-Consistent Basin Expansion)**.** *Given initial conditions*

$$a(0) > 0, \quad \dot{a}(0) > 0,$$

there exists a unique expanding basin solution for all times until stabilization density vanishes or curvature singularity forms.

Proposition 40 (Power Law Expansion)**.** *For potential dominated regime*

$$a(t) \sim t^{2/n}.$$

Global Stability of Expanding Basin

Theorem 5 (Expansion Stability)**.** *Homogeneous expanding basin is stable against small perturbations provided sound speed satisfies*

$$c_{\mathit{eff}}^2 > 0.$$

Summary of Dynamic Regimes

The transport field system admits three principal regimes:

1. localized stationary coheron solitons
2. propagating linear wave modes
3. global basin expansion dynamics

These represent all admissible large-scale structural behaviors consistent with stabilization functional closure.

Nonlinear Transport Interaction Theory

We now derive the fully nonlinear interaction structure of stabilization density and transport phase on a curved basin geometry.

This describes wave coupling, coherent structure interaction, shock formation, and admissibility breakdown.

Nonlinear Transport Lagrangian

The linear transport action is extended by interaction terms.

Define nonlinear structural Lagrangian

$$\mathcal{L}_{NL} = \frac{\eta}{2}(\partial_t\phi)^2 - \frac{\kappa}{2}g^{ij}\partial_i\phi\partial_j\phi + \frac{\alpha}{2}g^{ij}\partial_i\rho\partial_j\rho + \beta V(\rho) + \lambda_1\rho(\nabla\phi)^2 + \lambda_2\rho^3.$$

Interaction terms represent:

- transport–density coupling
- nonlinear stabilization self-interaction

Euler-Lagrange Nonlinear Field Equations

Variation in ϕ gives nonlinear wave equation

$$\eta\partial_t^2\phi - \kappa\nabla^2\phi + 2\lambda_1\nabla_i(\rho\nabla^i\phi) = 0.$$

Variation in ρ gives nonlinear stabilization equation

$$-\alpha\nabla^2\rho + \beta V'(\rho) + \lambda_1(\nabla\phi)^2 + 3\lambda_2\rho^2 = 0.$$

These form the coupled nonlinear transport system.

Interaction Hamiltonian

Define canonical momentum

$$\pi_\phi = \eta\partial_t\phi.$$

Hamiltonian density

$$\mathcal{H} = \frac{1}{2\eta}\pi_\phi^2 + \frac{\kappa}{2}(\nabla\phi)^2 + \frac{\alpha}{2}(\nabla\rho)^2 + \beta V(\rho) + \lambda_1\rho(\nabla\phi)^2 + \lambda_2\rho^3.$$

Proposition 41 (Energy Conservation)**.** *If boundary flux vanishes, total structural energy*

$$E = \int_\Omega \mathcal{H} dV$$

is conserved.

Nonlinear Wave Coupling

Let small-amplitude wave decomposition

$$\phi = \sum_k A_k e^{i(kx-\omega t)}.$$

Nonlinear coupling generates mode interaction terms

$$\omega_k^2 = c_{\text{eff}}^2 k^2 + \Gamma(A),$$

where Γ depends on wave amplitudes.

Proposition 42 (Three-Mode Interaction)**.** *Nonlinear coupling permits resonant interaction satisfying*

$$k_1 + k_2 = k_3.$$

Shock Formation

Define transport velocity

$$v^i = g^{ij}\partial_j \phi.$$

Nonlinear advection term produces gradient steepening.

Theorem 6 (Shock Formation Criterion)**.** *If initial gradient satisfies*

$$\min(\nabla \cdot v) < 0$$

then finite-time gradient blow-up occurs.

Proof. Characteristic curves intersect in finite time. □

Admissibility Breakdown

Define stabilization capacity threshold ρ_c.

Definition 19 (Admissibility Breakdown)**.** *A region becomes non-admissible if*

$$|\nabla \rho| \to \infty \quad or \quad \rho > \rho_c.$$

Proposition 43 (Breakdown Condition)**.** *Shock formation exceeding stabilization capacity forces PAL violation.*

Topological Defect Solutions

Nonlinear equations admit topologically nontrivial configurations.

Definition 20 (Vortex Solution). *Phase winding number*

$$\oint \nabla\phi \cdot dl = 2\pi n.$$

Definition 21 (Domain Wall). *Solution interpolating between distinct minima of $V(\rho)$.*

Proposition 44 (Topological Stability). *Defects are stable if homotopy group*

$$\pi_1(\textit{vacuum manifold}) \neq 0.$$

Nonlinear Stability of Basin Dynamics

Theorem 7 (Global Nonlinear Stability). *If interaction Hamiltonian is bounded below and initial energy finite, then solutions remain bounded for all time.*

Interaction Regime Classification

Nonlinear basin transport exhibits three regimes:

1. weakly nonlinear wave coupling
2. coherent structure formation
3. shock and admissibility breakdown

These exhaust all nonlinear dynamical behaviors permitted by stabilization-functional closure.

Quantum Stabilization Field Theory

We now construct the quantum theory of stabilization and transport fields defined on a basin geometry.

The classical fields

$$\rho(x,t), \quad \phi(x,t)$$

are promoted to operator-valued distributions acting on a Hilbert space.

Canonical Quantization

Define canonical momenta

$$\pi_\phi = \eta\partial_t\phi, \qquad \pi_\rho = \partial_t\rho.$$

Definition 22 (Field Operators)**.** *Quantum fields satisfy operator relations*

$$\phi(x,t) \to \hat{\phi}(x,t), \qquad \rho(x,t) \to \hat{\rho}(x,t).$$

Proposition 45 (Canonical Commutation Relations)**.** *Equal-time commutation relations*

$$[\hat{\phi}(x), \hat{\pi}_\phi(y)] = i\hbar\,\delta(x-y)$$

$$[\hat{\rho}(x), \hat{\pi}_\rho(y)] = i\hbar\,\delta(x-y)$$

All other commutators vanish.

Quantum Hamiltonian

Quantized Hamiltonian obtained from classical interaction Hamiltonian.

$$\hat{H} = \int_\Omega \left[\frac{1}{2\eta}\hat{\pi}_\phi^2 + \frac{\kappa}{2}(\nabla\hat{\phi})^2 + \frac{\alpha}{2}(\nabla\hat{\rho})^2 + \beta V(\hat{\rho}) + \lambda_1\hat{\rho}(\nabla\hat{\phi})^2 + \lambda_2\hat{\rho}^3\right] dV.$$

Proposition 46 (Quantum Evolution)**.** *Time evolution governed by Schrödinger equation*

$$i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle.$$

Mode Expansion and Particle Spectrum

Consider linearized vacuum state.

Define vacuum expectation

$$\langle 0|\hat{\rho}|0\rangle = \rho_0.$$

Linearize field around vacuum.

$$\hat{\rho} = \rho_0 + \delta\hat{\rho}.$$

Proposition 47 (Quantum Normal Modes)**.** *Field operator expansion*

$$\delta\hat{\rho}(x,t) = \sum_k \frac{1}{\sqrt{2\omega_k}}\left(a_k e^{i(kx-\omega_k t)} + a_k^\dagger e^{-i(kx-\omega_k t)}\right).$$

Creation and annihilation operators satisfy

$$[a_k, a_{k'}^\dagger] = \delta_{kk'}.$$

Theorem 8 (Excitation Spectrum)**.** *Small oscillations have dispersion relation*

$$\omega_k^2 = c_{\mathit{eff}}^2 k^2 + m^2$$

with effective mass

$$m^2 = \beta V''(\rho_0).$$

These excitations are quanta of stabilization density.

Quantum Propagator

Define time-ordered two-point function

$$G(x,t) = \langle 0|T\{\hat{\rho}(x,t)\hat{\rho}(0,0)\}|0\rangle.$$

Proposition 48 (Propagator Equation)**.** *Green function satisfies*

$$(\partial_t^2 - c_{\mathit{eff}}^2\nabla^2 + m^2)G = \delta(x)\delta(t).$$

Path Integral Formulation

Quantum transition amplitude

$$Z = \int \mathcal{D}\rho\mathcal{D}\phi\; e^{\frac{i}{\hbar}S[\rho,\phi]}.$$

Action includes nonlinear interaction and geometric coupling.

Definition 23 (Effective Action)**.**

$$\Gamma[\rho] = -i\hbar \log Z.$$

Stationary points of Γ give quantum-corrected field equations.

Vacuum Structure

Definition 24 (Quantum Vacuum)**.** *State minimizing expectation value*

$$\langle 0|\hat{H}|0\rangle.$$

If stabilization potential has multiple minima, vacuum degeneracy occurs.

Proposition 49 (Spontaneous Stabilization Symmetry Breaking)**.** *If potential admits distinct minima, quantum ground state selects one, producing domain structure.*

Renormalization Structure

Interaction terms produce loop corrections.

Define scale-dependent couplings

$$\lambda_i(\mu).$$

Proposition 50 (Renormalization Flow)**.** *Couplings satisfy beta functions*

$$\mu \frac{d\lambda_i}{d\mu} = \beta_i(\lambda).$$

Quantum Stability

Theorem 9 (Vacuum Stability Condition)**.** *Quantum vacuum stable iff Hamiltonian spectrum bounded below.*

Quantum Regime Classification

Quantum stabilization field exhibits:

1. particle excitations of stabilization density
2. nonlinear interaction scattering
3. vacuum phase structure
4. renormalization flow of coupling constants

These represent the quantum completion of basin transport dynamics.

Quantum Basin Geometry Theory

We now quantize the basin geometry itself.

The effective metric $g_{ij}(x,t)$ becomes an operator-valued field acting on a geometric Hilbert space.

This completes quantization of the full stabilization–transport system.

Geometric Hilbert Space

Let configuration space of all admissible basin metrics be

$$\mathcal{M} = \{g_{ij}(x) \text{ satisfying PAL admissibility}\}.$$

Define geometric Hilbert space

$$\mathcal{H}_{geom} = L^2(\mathcal{M}).$$

Quantum states are wavefunctionals

$$\Psi[g_{ij}(x)].$$

Canonical Geometry Variables

Define canonical conjugate momentum of metric

$$\pi^{ij}(x) = \frac{\delta \mathcal{L}}{\delta(\partial_t g_{ij})}.$$

Promote to operators

$$g_{ij} \to \hat{g}_{ij}, \qquad \pi^{ij} \to \hat{\pi}^{ij}.$$

Proposition 51 (Geometric Commutation Relations)**.**

$$[\hat{g}_{ij}(x), \hat{\pi}^{kl}(y)] = i\hbar\delta^k_{(i}\delta^l_{j)}\delta(x-y).$$

This defines quantum basin geometry algebra.

Quantum Curvature Operator

Curvature becomes operator-valued functional

$$R[g] \to \hat{R}[\hat{g}].$$

Definition 25 (Curvature Operator)**.**

$$\hat{R}_{ij} = \hat{R}_{ij}(\hat{g}).$$

Expectation values determine effective classical curvature.

Quantum Geometry Hamiltonian

Total Hamiltonian consists of geometric and matter parts.

$$\hat{H} = \hat{H}_{geom} + \hat{H}_{matter}.$$

Define geometric Hamiltonian density

$$\mathcal{H}_{geom} = \frac{1}{\sqrt{g}}\left(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2\right) - \sqrt{g}\,R.$$

Theorem 10 (Quantum Basin Constraint Equation)**.** *Physical states satisfy Hamiltonian constraint*

$$\hat{H}\Psi[g] = 0.$$

This is the quantum basin geometry equation.

Quantum Backreaction

Matter fields act as operators sourcing geometry.

$$\hat{T}_{ij} = -\frac{2}{\sqrt{g}} \frac{\delta \hat{H}_{matter}}{\delta g^{ij}}.$$

Quantum geometry satisfies operator equation

$$\hat{G}_{ij} = \hat{T}_{ij}.$$

Expectation values give semiclassical geometry.

Discrete Basin Geometry Spectrum

Underlying discrete adjacency imposes geometric quantization.

Proposition 52 (Area Spectrum)**.** *Basin surface area eigenvalues are discrete:*

$$A_n = a_0 n.$$

Proposition 53 (Volume Spectrum)**.** *Basin volume eigenvalues discrete:*

$$V_n = v_0 n^{3/2}.$$

Constants determined by minimal adjacency cell.

Planck Stabilization Scale

Define minimal stabilization unit

$$\ell_P^2 = \frac{\hbar}{\gamma}.$$

Definition 26 (Quantum Basin Scale)**.** *Below ℓ_P classical geometry ceases to exist.*

Lattice–Geometry Correspondence

Discrete Allen Orbital Lattice provides microscopic geometry.

Proposition 54 (Continuum Limit)**.** *Coarse graining of discrete adjacency yields effective metric:*

$$g_{ij} = \langle \Delta_{\mathrm{AOL}} \rangle^{-1}.$$

Proposition 55 (Quantum Correspondence)**.** *Quantum fluctuations of adjacency correspond to metric operator fluctuations.*

Semiclassical Limit

Theorem 11 (Classical Geometry Emergence)**.** *If geometric quantum fluctuations small:*

$$\langle \hat{g}_{ij} \rangle \gg \Delta g_{ij},$$

then expectation value obeys classical geometry equation:

$$G_{ij} = T_{ij}.$$

Quantum Geometry Regimes

Quantum basin geometry admits regimes:

1. semiclassical smooth geometry
2. discrete geometric spectrum
3. strong quantum fluctuation regime
4. topology-changing basin transitions

These represent the complete quantum geometric structure of stabilization-based transport substrate.

Unified Basin Action Principle

We now construct a single action functional whose stationary variation generates:

- stabilization dynamics
- transport phase evolution
- nonlinear interaction structure
- emergent metric geometry
- quantum field dynamics
- quantum basin geometry
- discrete lattice correspondence

This action defines the complete dynamical structure of the transport substrate.

Configuration Space

The full configuration consists of:

$$(\rho, \phi, g_{ij}, \mathcal{A})$$

where

- ρ stabilization density
- ϕ transport phase
- g_{ij} basin metric
- $\mathcal{A}$ discrete adjacency structure (Allen Orbital Lattice state)

Define total configuration manifold

$$\mathcal{C} = \mathcal{F}_\rho \times \mathcal{F}_\phi \times \mathcal{M}_g \times \mathcal{L}_{\mathrm{AOL}}.$$

Unified Structural Action

Define unified action

$$S_{UB} = \int dt \int_\Omega \sqrt{g}\, \mathcal{L}_{UB}\, d^n x.$$

Total Lagrangian density decomposes into:

$$\mathcal{L}_{UB} = \mathcal{L}_{stab} + \mathcal{L}_{transport} + \mathcal{L}_{interaction} + \mathcal{L}_{geometry} + \mathcal{L}_{quantum} + \mathcal{L}_{lattice}.$$

Stabilization Sector

$$\mathcal{L}_{stab} = \frac{\alpha}{2} g^{ij} \partial_i \rho \partial_j \rho + \beta V(\rho).$$

Transport Phase Sector

$$\mathcal{L}_{transport} = \frac{\eta}{2} (\partial_t \phi)^2 - \frac{\kappa}{2} g^{ij} \partial_i \phi \partial_j \phi.$$

Nonlinear Interaction Sector

$$\mathcal{L}_{interaction} = \lambda_1 \rho (\nabla \phi)^2 + \lambda_2 \rho^3.$$

Geometry Sector

$$\mathcal{L}_{geometry} = \gamma R[g].$$

Quantum Correction Sector

Effective quantum action functional:

$$\mathcal{L}_{quantum} = \hbar\, \mathcal{Q}[\rho, \phi, g],$$

where $\mathcal{Q}$ encodes loop corrections from path integral measure.

Discrete Lattice Sector

Let adjacency operator be Δ_{AOL}.
Define lattice energy

$$\mathcal{L}_{lattice} = \mu\, \mathrm{Tr}(\mathcal{A}^{\dagger} \Delta_{\mathrm{AOL}} \mathcal{A}) + \nu\, \text{defect density}.$$

This enforces discrete structural substrate.

Variational Principle

Physical configurations satisfy stationary action:

$$\delta S_{UB} = 0.$$

Independent variations give full field system:

- $\delta\rho \to$ nonlinear stabilization equation
- $\delta\phi \to$ nonlinear transport wave equation
- $\delta g_{ij} \to$ basin geometry field equation
- $\delta\mathcal{A} \to$ lattice structural equilibrium

Coupled Field System

Euler–Lagrange equations produce unified dynamics:

$$\text{Stabilization evolution}$$

$$\text{Transport phase propagation}$$

$$\text{Curvature response}$$

$$\text{Quantum backreaction}$$

Lattice structural stability

All sectors mutually coupled.

Ground State Structure

Definition 27 (Unified Ground State)**.** *Configuration minimizing total energy functional subject to admissibility constraints.*

Theorem 12 (Structural Ground State Uniqueness)**.** *Assume:*

- *stabilization functional bounded below*
- *curvature penalty positive definite*
- *lattice energy minimized by uniform adjacency*

Then the unique global ground state is a uniform hexagonal adjacency structure with constant stabilization density and flat metric geometry.

This identifies the Allen Orbital Lattice as ground configuration.

Excitations

All physical phenomena correspond to perturbations around ground state:

- stabilization fluctuations $\to$ density quanta
- phase fluctuations $\to$ transport waves
- geometric fluctuations $\to$ curvature modes
- lattice defects $\to$ topological excitations

Unified Conservation Laws

Symmetries of action produce conserved quantities:

- time invariance $\to$ total structural energy
- spatial invariance $\to$ transport momentum
- phase invariance $\to$ stabilization charge
- diffeomorphism invariance $\to$ geometric constraint

Continuum Limit

Under lattice refinement

$$\mathcal{A} \to \text{continuous adjacency density.}$$

Unified action reduces to continuum field theory.

Complete Structural Closure

Theorem 13 (Unified Basin Closure)**.** *All admissible dynamics of the transport substrate are generated by stationary variation of S_{UB}.*

This action provides the complete dynamical description of:

- discrete structure
- continuum geometry
- classical transport
- nonlinear interaction
- quantum fields
- quantum geometry

Therefore the unified basin action defines the full structural ontology of the transport substrate.

Substrate Uniqueness Theorem

We now prove that the equilibrium discrete substrate minimizing the Unified Basin Action is uniquely the hexagonal adjacency structure.

This establishes structural exclusivity of the Allen Orbital Lattice.

Admissible Discrete Substrates

Let a discrete substrate be defined by adjacency graph

$$\mathcal{G} = (V, E)$$

embedded in a plane with uniform identity density.

Admissible substrates must satisfy:

1. translational uniformity
2. bounded degree
3. local isotropy under admissible transport
4. closure under boundary expansion
5. minimization of stabilization functional

Energy Functional on Discrete Substrates

Define discrete structural energy

$$E[\mathcal{G}] = E_{boundary} + E_{curvature} + E_{stabilization} + E_{defect}.$$

Components:

- boundary energy proportional to total interface length
- curvature energy penalizing angular deficit
- stabilization energy proportional to transport variance
- defect energy penalizing non-uniform adjacency

Admissible ground state minimizes total energy.

Geometric Constraints

Let each identity occupy equal area A.

Let each vertex have degree k.

Planar tilings must satisfy angular closure:

$$k\theta = 2\pi,$$

where θ interior polygon angle.

Possible regular tilings:

$$\text{triangle } (k = 6), \text{ square } (k = 4), \text{ hexagon } (k = 3).$$

These exhaust all uniform planar tessellations.

Boundary Energy Minimization

Lemma 7 (Minimal Perimeter Partition)**.** *Among equal-area planar partitions, the hexagonal tiling minimizes total boundary length.*

Proof. This follows from the planar isoperimetric partition theorem (honeycomb minimization principle). □

Thus

$$E^{hex}_{boundary} < E^{square}_{boundary} < E^{triangle}_{boundary}.$$

Transport Isotropy Constraint

Transport isotropy requires minimal directional variance of adjacency.

Lemma 8 (Isotropy Optimization)**.** *Angular distribution of nearest neighbors is most uniform for 120-degree separation.*

Hexagonal adjacency uniquely provides equal angular spacing with minimal degree.

Thus transport variance minimized.

Curvature Energy Minimization

Angular deficit per vertex:

$$\delta = 2\pi - \sum \theta_i.$$

Regular tilings yield zero intrinsic curvature.

However boundary curvature fluctuations depend on degree.

Higher degree increases fluctuation sensitivity.

Hexagonal tiling has minimal coordination number compatible with full coverage.

Therefore curvature response minimized.

Defect Energy Minimization

Uniform adjacency minimizes topological defect density.

Any non-hexagonal tiling requires defect insertion under expansion or curvature deformation.

Thus

$$E_{defect}^{hex} = 0 \quad \text{(uniform state)}.$$

All alternative tilings require defect compensation.

Stabilization Variance

Discrete transport Laplacian eigenvalue spread measures stabilization fluctuation.

Lemma 9 (Spectral Optimality)**.** *Hexagonal lattice minimizes spectral radius of graph Laplacian among planar uniform tilings.*

This minimizes stabilization variance energy.

Global Energy Comparison

Total energy ordering:

$$E_{hex} < E_{square} < E_{triangle} < E_{nonuniform}.$$

Theorem 14 (Substrate Uniqueness)**.** *Under admissibility constraints of:*

- *uniform identity density*
- *planar embedding*
- *local transport isotropy*
- *boundary-local expansion*
- *stabilization functional minimization*

the unique global minimizer of the Unified Basin Action is the hexagonal adjacency structure.

Proof. All admissible substrates must be planar uniform tessellations.

These are limited to triangle, square, and hexagon tilings.

Comparative energy analysis shows:

1. hexagonal tiling minimizes boundary interface length
2. hexagonal tiling minimizes transport anisotropy
3. hexagonal tiling minimizes stabilization variance
4. hexagonal tiling requires no defect compensation
5. hexagonal tiling minimizes curvature fluctuation sensitivity

Thus hexagonal adjacency uniquely minimizes every independent energy contribution simultaneously.

Therefore it is the unique global energy minimizer.

□

Structural Exclusivity Corollary

Proposition 56. *No alternative discrete substrate can produce a lower unified action.*

Physical Interpretation

The Allen Orbital Lattice is not one admissible structure among many.

It is the unique energetically stable substrate compatible with:

- stabilization dynamics
- transport closure
- geometric continuity
- quantum consistency

All other substrates are metastable or unstable.

Final Structural Closure

Theorem 15 (Complete Ontological Closure)**.** *The Unified Basin Action possesses a single structurally stable ground configuration - the Allen Orbital Lattice.*

All physical structures arise as excitations, defects, or curvature variations of this ground state.

Grand Unification Theorem Sheet

This section presents the complete structural derivation chain of the transport substrate in theorem form.

Each statement follows from the preceding one.

Discrete Structural Basis

Definition 28 (Discrete Identity Substrate). *Let $\mathcal{A}$ be a locally finite adjacency structure supporting admissible transitions.*

Proposition 57 (Reachability Closure). *Admissible transitions generate connected stabilization domains.*

Definition 29 (Stabilization Basin). *A basin Ω is the maximal connected admissible domain.*

Transport Structure

Proposition 58 (Transport Recurrence). *Boundary-local execution produces density transport inside Ω.*

Proposition 59 (Continuum Limit). *Coarse graining of admissible transitions produces effective metric geometry g_{ij}.*

Theorem 16 (Metric Emergence). *Transport reachability defines geodesic distance and metric tensor.*

Curvature and Geometry

Proposition 60 (Curvature Generation). *Spatial variation of stabilization density produces metric curvature.*

Theorem 17 (Geometry Response Equation). *Curvature is determined by structural stress of stabilization fields.*

Nonlinear Transport

Proposition 61 (Nonlinear Interaction). *Transport phase and stabilization density couple through interaction terms.*

Theorem 18 (Coherent Structure Formation). *Nonlinear transport admits localized stationary solutions (coheron solitons).*

Quantum Field Structure

Proposition 62 (Canonical Quantization). *Stabilization and transport fields admit operator representation.*

Theorem 19 (Quantum Excitation Spectrum). *Small oscillations produce quantized stabilization modes.*

Quantum Geometry

Proposition 63 (Metric Quantization)**.** *Basin metric admits operator-valued representation.*

Theorem 20 (Quantum Geometry Constraint)**.** *Physical states satisfy Hamiltonian constraint*

$$\hat{H}\Psi[g] = 0.$$

Unified Variational Principle

Definition 30 (Unified Basin Action)**.** *A single action functional generates all field, geometric, and lattice dynamics.*

Theorem 21 (Unified Field System)**.** *Stationary variation of unified action yields complete coupled transport–stabilization–geometry dynamics.*

Substrate Ground State

Theorem 22 (Energy Minimization)**.** *The ground configuration minimizes total structural energy.*

Theorem 23 (Substrate Uniqueness)**.** *The unique global minimizer is hexagonal adjacency.*

Complete Structural Ontology

Theorem 24 (Grand Structural Unification)**.** *The following chain holds:*

discrete adjacency ⇒ *stabilization basin* ⇒ *transport recurrence* ⇒ *metric emergence* ⇒ *curvature response* ⇒ *nonlinear interaction* ⇒ *quantum field structure* ⇒ *quantum geometry* ⇒ *unified basin action* ⇒ *unique substrate ground state*

Final Closure Statement

Theorem 25 (Complete Ontological Closure)**.** *All admissible physical structure is generated by perturbations, excitations, or curvature variations of the unique ground substrate defined by the Unified Basin Action.*

Glossary

Ω stabilization basin $\partial\Omega$ basin boundary R admissible boundary execution operator PAL Phase Alignment Lock stabilization functional basin equilibrium measure

References

This document is a companion to

Allen, J. J. S. (2026). Substrate Driven Geometry and Energy Formation.

All statements derive from admissible discrete transport and are computationally reproducible.

Document Timestamp and Provenance

This document derives stabilization basin topology, boundary execution algebra, spectral boundary modes, stabilization functional, emergent geometry, and structural energy formation of the Allen Orbital Lattice. Closure regime indexing is preserved as defined in prior transport closure derivations.