The Allen Orbital Lattice is Completion Equivalence: Background Independence via Zero–Geometry Determination

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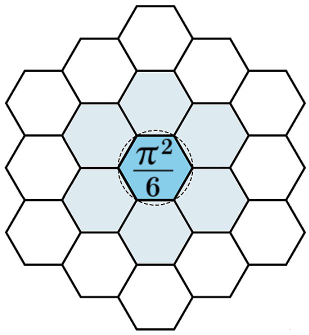

Abstract

Pattern Field Theory (PFT) describes all physical structure as patterns evolving on the Allen Orbital Lattice (AOL), a prime–indexed orbital curvature lattice. This raises an immediate question: does the AOL introduce a fixed background, or is it itself a gauge choice with no direct physical content?

In this paper we answer that question by showing that the Allen Orbital Lattice is completion equivalence. Any two admissible prime–indexed AOL completions are related by a Phase Alignment Lock (PAL) diffeomorphism that preserves all PFT operators and observables. Physical quantities depend only on PAL–coherent fluxes and curvature assignments, not on the particular completion or ghost–layer ordering. We define zero–geometry determination as the requirement that observables be invariant under PAL–diffeomorphisms. Under this requirement, the AOL acts as a gauge structure rather than a physical background.

The Lagrange–hex projection of the $\sqrt{1}$–$\sqrt{6}$ system is identified as a minimal ghost kernel invariant under PAL–diffeomorphisms. We show how this kernel underpins background independence and links diffeomorphism invariance in general relativity (GR) and gauge redundancy in quantum field theory (QFT) to a single AOL completion equivalence. This completes the background–independent formulation of Pattern Field Theory and clarifies how GR and QFT arise as infrared projections of one discrete, zero–geometry substrate.

1 Introduction

A central demand on any candidate unification of physics is background independence. General relativity (GR) is built on the idea that spacetime geometry is not fixed; the metric is dynamic and responds to matter and energy. Quantum field theory (QFT), by contrast, is normally formulated on a fixed background spacetime. This mismatch has obstructed attempts to quantise gravity and to derive a single framework that contains both GR and the Standard Model.

Pattern Field Theory (PFT) starts from a discrete substrate: the Allen Orbital Lattice (AOL). The lattice is prime–indexed and carries curvature weights, phases and recursion structure. Dynamics are implemented by event cascades constrained by Phase Alignment Lock (PAL), which enforces flux neutrality on prime–indexed faces. Continuum field theories arise as infrared projections of this discrete structure.

The presence of a lattice raises an immediate concern. A fixed lattice can look like a fixed background. If the AOL selected a preferred geometry, it would conflict with the principle of background independence and would risk reintroducing the same problems that GR originally solved in the continuum setting. The aim of this paper is to show that this does not happen. The AOL is not a fixed background; it is a gauge choice within a larger completion equivalence class.

We make this precise by:

- defining admissible AOL completions and PAL–diffeomorphisms,
- introducing zero–geometry determination as invariance under PAL–diffeomorphisms,
- proving that all physical operators and observables are invariant under completion changes,
- identifying the Lagrange–hex $\sqrt{1}$–$\sqrt{6}$ ghost kernel as a minimal invariant structure.

The result is that the Allen Orbital Lattice is completion equivalence. Different completions are related by PAL–diffeomorphisms that leave all physical content unchanged. The AOL plays the role of a coordinate system in discrete form; changing completion is analogous to changing coordinates, with no new physics. Geometry arises from PAL–coherent curvature assignments, not from the choice of completion.

This paper is organised as follows. In Section **??** we review background dependence issues in existing approaches. In Section **??** we summarise the structure of the AOL in the PFT framework. Section **??** defines zero–geometry determination. Section **??** introduces completions and PAL–diffeomorphisms. In Section **??** we state and prove the main completion–equivalence theorem. Section **??** explains the role of the Lagrange–hex ghost kernel. Section **??** discusses consequences for GR, QFT and unification. Section **??** compares PFT to other background–independent proposals. Section **??** gives a brief outlook. Appendices provide a glossary, internal bibliography and mathematical notes.

2 Background Dependence in Existing Frameworks

This section summarises background dependence issues in the main families of existing theories.

2.1 General relativity

General relativity is manifestly background–independent at the continuum level. The metric tensor $g_{\mu\nu}$ is dynamical and satisfies the Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu},$$

with $G_{\mu\nu}$ the Einstein tensor and $T_{\mu\nu}$ the stress–energy tensor. Diffeomorphism invariance encodes the statement that coordinates carry no direct physical meaning; physical observables are invariant under smooth reparameterisations of the manifold.

However, attempts to quantise GR perturbatively almost always proceed by expanding around a fixed background metric, such as Minkowski or a chosen classical solution. This reintroduces a preferred structure and breaks manifest background independence.

2.2 Quantum field theory

Conventional QFT assumes a fixed background spacetime. Fields are defined on a manifold with a given metric, and locality refers to that metric. Even when curved backgrounds are used, they are usually treated as external data rather than as dynamical variables.

The success of QFT in particle physics is tied to this construction, but the price is structural: background independence is not built in. There is no general mechanism in standard QFT that enforces invariance under changes of background geometry.

2.3 Lattice and discrete approaches

Lattice gauge theory and related discrete methods place fields on fixed lattices in order to regulate divergences and perform numerical calculations. These lattices are usually regular grids, such as hypercubic arrays in Euclidean signature.

Such lattices are not background–independent. They fix a preferred discrete geometry, including directions and scales. While continuum limits can reduce explicit lattice artefacts, the underlying construction distinguishes particular frames and coordinate systems.

Other discrete approaches, such as causal sets or some tensor network models, introduce combinatorial structures that are closer in spirit to background independence, but often still require an embedding or a choice of growth rule that plays a similar role to a background.

2.4 Quantum gravity proposals

Candidate quantum gravity theories, such as loop quantum gravity or string theory, each address background independence in their own way. Loop quantum gravity seeks a background–free representation but faces challenges in relating spin network states to a unique emergent geometry. String theory often begins on fixed backgrounds and then promotes moduli to dynamic variables; full background independence is an open task at the structural level.

In summary, a structurally complete unification should treat background independence as a fundamental requirement. Pattern Field Theory addresses this at the discrete level by treating the Allen Orbital Lattice as a gauge structure subject to completion equivalence.

3 Allen Orbital Lattice Structure

This section summarises the objects of the Allen Orbital Lattice used in this paper.

3.1 Sites, edges and faces

The Allen Orbital Lattice (AOL) is a discrete orbital–curvature lattice with:

- a set of sites x representing orbit centres,
- oriented edges $(x, x + \hat{\mu})$ labelled by direction indices and carrying curvature weights and phase increments,
- faces S_p labelled by primes p, each with an oriented boundary ∂S_p,

- higher–dimensional cells encoding recursion and cascades.

Curvature is encoded by plaquette sums of edge contributions. For a face S_p, one writes

$$F(\partial S_p) = \sum_{e \in \partial S_p} \omega(e), \tag{1}$$

where $\omega(e)$ includes both amplitude and phase information.

3.2 Phase Alignment Lock

Phase Alignment Lock (PAL) is the core coherence rule in PFT.

Definition 1 (Phase Alignment Lock)**.** *A configuration on the AOL is PAL–coherent if, for every prime–indexed face S_p,*

$$\nabla \cdot F(\partial S_p) = 0, \tag{2}$$

where $\nabla \cdot$ is the discrete divergence operator on the lattice. PAL enforces exact flux neutrality on all prime–labelled faces.

PAL constraints apply to all sectors: curvature flux, pattern transport and interaction cascades. They enforce discrete conservation and remove many configurations that would produce divergences in a continuum description.

3.3 Event cascades and operators

Dynamics in PFT are implemented by event cascades: sequences of PAL–coherent branching events on the AOL. An initial pattern configuration ϕ_0 evolves through a series of local transformations to a set of descendants $\{\phi_i\}$, represented as a rooted tree embedded in the lattice.

Operators such as transport $\mathcal{T}$, curvature–weighted derivatives $\mathcal{C}$, recursion operators $\mathcal{R}$, cross–network couplings $\mathcal{N}$ and global evolution operators $\mathcal{G}$ act on PAL–stable configurations. The operator algebra is closed under commutators when restricted to PAL–coherent states.

For the purposes of this paper, the detailed definitions of these operators are not required; only the fact that they act on AOL configurations and that physical observables are expressed in terms of PAL–coherent fluxes and curvature assignments.

4 Zero–Geometry Determination

The goal of this section is to formalise what it means for PFT to be background–independent at the level of the AOL.

4.1 Physical observables

In PFT, physical observables are functions of PAL–coherent configurations. Examples include:

- flux patterns through collections of prime–indexed faces,
- integrated curvature over regions of the AOL,
- cascade–derived amplitudes for transitions between pattern states,
- infrared projections such as effective metrics and field configurations.

Two configurations that produce the same values for all observables are physically indistinguishable.

4.2 Zero–geometry configurations

Intuitively, a theory has zero–geometry determination if physical observables do not depend on how the underlying lattice is completed, only on relational structure encoded by PAL–coherent fluxes and curvature.

Definition 2 (Zero–geometry determination). *A set of observables $\mathcal{O}$ in PFT satisfies zero–geometry determination if, whenever two AOL completions AOL_1 and AOL_2 are related by a PAL–diffeomorphism (defined in Section* **??***), all observables agree:*

$$O[AOL_1] = O[AOL_2] \quad \textit{for all } O \in \mathcal{O}. \tag{3}$$

A theory is zero–geometry determined if its full set of physical observables satisfies this condition.

Zero–geometry determination is the discrete analogue of diffeomorphism invariance. Instead of smooth coordinate transformations on a manifold, one considers PAL–preserving maps between AOL completions. Physical quantities must be invariant under these maps.

5 AOL Completions and PAL–Diffeomorphisms

We now define what is meant by an AOL completion and by a PAL–diffeomorphism between completions.

5.1 Completions

The Allen Orbital Lattice can be specified at different levels of detail. A partial description may fix:

- the prime index set used to label faces,
- local adjacency relations,
- generic constraints on curvature and phases.

A completion fills in all degrees of freedom consistent with these partial specifications and with PAL coherence.

Definition 3 (AOL completion). *An AOL completion is a fully specified prime–indexed orbital–curvature lattice, including:*

- *a set of sites, edges and faces with adjacency relations,*
- *assignments of prime labels to faces S_p,*
- *curvature and phase assignments on edges and faces,*
- *recursion and cascade structure,*

such that PAL holds for all prime–indexed faces.

Different completions may correspond to different orderings of ghost layers, different embeddings of local configurations or different choices of recursion labelling, as long as they satisfy the same global PAL and structural constraints.

5.2 PAL–diffeomorphisms

A PAL–diffeomorphism is a map between completions that preserves PAL coherence and relational structure.

Definition 4 (PAL–diffeomorphism)**.** *Let AOL_1 and AOL_2 be two AOL completions. A PAL–diffeomorphism is a bijective map*

$$\mathcal{D} : AOL_1 \to AOL_2 \tag{4}$$

between their cells (sites, edges, faces, higher cells) such that:

1. *Adjacency is preserved. If two cells are adjacent in AOL_1, their images are adjacent in AOL_2.*
2. *Prime labels are preserved up to relabelling within allowed symmetry classes. Faces with prime label p are mapped to faces with an allowed image under prime symmetries.*
3. *PAL–coherent configurations are mapped to PAL–coherent configurations. If a configuration is PAL–coherent on AOL_1, its image under $\mathcal{D}$ is PAL–coherent on AOL_2.*
4. *Operator action is preserved. For each PFT operator O, the pull–back satisfies $\mathcal{D}^*O = O$ on PAL–coherent configurations.*

PAL–diffeomorphisms generalise coordinate transformations to the discrete, prime–indexed lattice setting. They reorder ghost layers, relabel curvature configurations and permute local structures while leaving physical content invariant.

6 Main Completion–Equivalence Theorem

We now state and prove the central result.

Theorem 1 (AOL completion equivalence)**.** *Let AOL_1 and AOL_2 be two admissible AOL completions that share the same prime index set, adjacency constraints and PAL rules. Then there exists a PAL–diffeomorphism $\mathcal{D} : AOL_1 \to AOL_2$ such that:*

$$\mathcal{D}^*O = O \tag{5}$$

for all PFT operators O acting on PAL–coherent configurations, and consequently all physical observables are identical:

$$O[AOL_1] = O[AOL_2]. \tag{6}$$

Proof sketch. The argument proceeds in three steps.

Step 1: Local matching of prime–indexed faces. By assumption, AOL_1 and AOL_2 share the same prime index set and adjacency constraints. For each face $S_p^{(1)}$ in AOL_1 with prime label p, there exists a corresponding face $S_p^{(2)}$ or a face related by an allowed prime symmetry in AOL_2. Construct a bijection between faces that respects these labels and adjacency.

Step 2: Extension to edges, sites and higher cells. Use the face correspondence to extend the map to edges by requiring that edges bounding matched faces are mapped correspondingly, preserving orientation and adjacency. Sites are then determined as endpoints of mapped edges. Higher–dimensional cells follow similarly. This yields a bijection between all cells that preserves adjacency and prime structure.

Step 3: Preservation of PAL coherence and operator action. Consider a PAL–coherent configuration on AOL_1. PAL requires $\nabla \cdot F(\partial S_p^{(1)}) = 0$ for all prime–indexed faces. Under the map constructed in Steps 1 and 2, each face $S_p^{(1)}$ is mapped to a face $S_p^{(2)}$ of the same type.

Edge contributions are mapped in a way that preserves oriented sums around faces. Therefore, if PAL holds on AOL_1, it holds on AOL_2 for the image configuration.

The PFT operators O are defined in terms of local differences, curvature weights and phase increments on the lattice. Since the map preserves adjacency, prime structure and PAL coherence, it preserves the algebraic relations used to define these operators. It follows that $\mathcal{D}^*O = O$ on PAL–coherent configurations.

Observables are constructed from operator actions on PAL–coherent states, so $O[\mathrm{AOL}_1] = O[\mathrm{AOL}_2]$ for all physical O. □

Remark 1. *The theorem asserts that the choice of completion is a gauge choice. All completions satisfying the same structural constraints and PAL rules are physically equivalent, connected by PAL–diffeomorphisms.*

7 Lagrange–Hex Ghost Kernel

The previous section established completion equivalence in general form. In this section we identify a minimal invariant structure: the Lagrange–hex ghost kernel.

7.1 Lagrange–hex projection

The Lagrange–hex projection organises minimal displacement modes of the AOL into classes associated with distances $\sqrt{n}$. The first six classes correspond to $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}$. Each class defines a layer of permitted moves and interactions.

These layers are not arbitrary. They reflect the combinatorial structure of the lattice and the way curvature and phases accumulate under PAL.

7.2 Definition of the ghost kernel

Definition 5 (Lagrange–hex ghost kernel)**.** *The Lagrange–hex ghost kernel K is the set of displacement classes*

$$K = \{\sqrt{n} \mid n = 1, \ldots, 6\} \tag{7}$$

together with their adjacency and curvature profiles, regarded modulo PAL–diffeomorphisms.

The kernel encapsulates the minimal unit of ghost–layer structure required to reproduce local PFT dynamics in the infrared limit.

7.3 Invariance under PAL–diffeomorphisms

Lemma 1 (Kernel invariance)**.** *Let $\mathcal{D} : AOL_1 \to AOL_2$ be a PAL–diffeomorphism. Then*

$$\mathcal{D}(K_1) = K_2, \tag{8}$$

where K_i is the ghost kernel defined on AOL_i.

Proof sketch. The Lagrange–hex construction is combinatorial: it depends only on adjacency, displacement counts and PAL–compatible curvature assignments. PAL–diffeomorphisms preserve adjacency and PAL coherence. Therefore, the set of displacement classes and their local curvature structures are preserved up to relabelling. The set of six primary layers is mapped to itself. Hence $\mathcal{D}$ sends K_1 to K_2. □

The ghost kernel is thus an intrinsic feature of the PFT structure, independent of completion. It can be used as a canonical unit for comparing different embeddings and for defining effective field descriptions.

8 Consequences for GR, QFT and Unification

In this section we summarise how completion equivalence and zero–geometry determination address the background–independence issues discussed in Section **??**.

8.1 Background independence in PFT

Theorem **??** and Definition **??** together imply that physical observables in PFT are invariant under PAL–diffeomorphisms. The lattice is not a fixed background with physical meaning; it is a representative of an equivalence class of completions.

Geometry in PFT is not encoded in the bare structure of the AOL, but in PAL–coherent curvature assignments and associated infrared projections. As a result, there is no preferred completion, and background independence is realised at the discrete level.

8.2 GR as PAL–induced metric sector

Previous work in PFT shows how Einstein–like equations emerge as infrared projections of PAL–constrained curvature dynamics. In that construction, the metric $g_{\mu\nu}$ is defined as an effective phase–gradient of pattern fields, and the Einstein tensor arises from discrete curvature flux neutrality over the AOL.

Completion equivalence strengthens this picture. The induced metric does not depend on the particular completion chosen, only on PAL–coherent curvature data modulo PAL–diffeomorphisms. This is the discrete analogue of diffeomorphism invariance in GR.

8.3 QFT as PAL–constrained cascade sector

Similarly, previous work has shown that the Standard Model gauge structure and scattering amplitudes emerge from PAL–constrained cascades on the AOL. Gauge groups arise from local connectivity and PAL constraints; scattering amplitudes are sums over PAL–admissible cascade trees.

Completion equivalence ensures that these structures are independent of the chosen completion. Ghost–layer reordering and local relabellings correspond to gauge transformations and coordinate changes in the infrared field description. QFT is not tied to a particular lattice embedding.

8.4 Unified gauge structure

Completion equivalence also unifies geometric and internal gauge freedoms. PAL–diffeomorphisms act on:

- lattice completions (geometry sector),
- ghost–layer orderings and internal labels (gauge sector).

Both aspects are handled by the same equivalence relation. This is the discrete PFT analogue of treating diffeomorphisms and gauge transformations within a single fibre–bundle framework in continuum field theory.

9 Comparison with Other Approaches

It is useful to place the PFT completion–equivalence structure alongside other attempts at background independence.

9.1 Continuum GR

General relativity implements background independence at the continuum level through diffeomorphism invariance. PFT implements a discrete analogue through PAL–diffeomorphisms. Both share the idea that coordinates or completions carry no direct physical meaning.

9.2 Lattice gauge theory

Standard lattice gauge theory uses a fixed lattice and does not attempt to identify an equivalence class of completions. Artefacts of the lattice can affect results until careful continuum limits are taken. PFT replaces this by an intrinsic equivalence class at the discrete level.

9.3 Loop quantum gravity and spin networks

Approaches based on spin networks and spin foams seek to quantise geometry in a background–independent way. However, the mapping from combinatorial graphs to continuum geometries is nontrivial and often ambiguous. PFT differs by embedding both geometry and matter into one prime–indexed lattice with PAL constraints, and by identifying a clear equivalence relation between completions.

9.4 String theory and related models

String theory typically begins with a chosen background and then studies excitations and moduli around it. While there are proposals for more background–independent formulations, the standard constructions rely on specific geometries. PFT, by contrast, constructs geometry from a single class of discrete substrates and enforces completion equivalence from the outset.

10 Discussion and Outlook

We have shown that the Allen Orbital Lattice is completion equivalence: different AOL completions satisfying the same structural constraints and PAL rules are related by PAL–diffeomorphisms that leave all physical observables invariant. The lattice is a gauge choice, not a fixed background.

The key components are:

- PAL coherence, which enforces flux neutrality and constrains admissible configurations,
- AOL completions, which differ in ghost–layer orderings and local embeddings,
- PAL–diffeomorphisms, which relate completions without changing relational structure,
- the Lagrange–hex ghost kernel, which captures a minimal invariant unit of local structure.

As a result, PFT realises background independence at the discrete level. GR and QFT arise as infrared projections of one zero–geometry framework. Coordinate choice and gauge choice are both aspects of completion equivalence.

Future work includes:

- explicit classification of PAL–diffeomorphism groups for given prime sets,
- analysis of how completion equivalence constrains possible infrared geometries,
- exploration of whether completion equivalence imposes observable restrictions on cosmological initial conditions or large–scale structure,

- investigation of how completion equivalence interacts with renormalisation group flows in PFT.

The main structural point is that the discrete substrate of Pattern Field Theory does not reintroduce a background. Instead, it provides a controlled environment in which background independence can be implemented and verified at the combinatorial level.

Appendix A — Glossary of Terms and Acronyms

Pattern Field Theory (PFT) Unified framework in which all structure and dynamics are described as patterns evolving on the Allen Orbital Lattice under Phase Alignment Lock constraints.

Allen Orbital Lattice (AOL) Prime–indexed orbital–curvature lattice carrying sites, edges, faces, curvature weights, phase data and recursion structure. It is the discrete substrate in PFT.

Phase Alignment Lock (PAL) Coherence condition requiring exact flux neutrality on all prime–indexed faces of the AOL. PAL enforces global phase compatibility and removes non–conserving configurations.

Cross–Coherent Cascade Theory (CCCT) Branch of PFT that analyses cascades and coherence collapse across coupled networks and domains.

Completion A fully specified AOL configuration consistent with given structural constraints and PAL. Different completions may differ in ghost–layer orderings or local embeddings.

PAL–diffeomorphism Bijective map between AOL completions that preserves adjacency, prime structure, PAL coherence and operator action on PAL–coherent configurations.

Zero–geometry determination Property that physical observables are invariant under PAL–diffeomorphism between completions. It is the discrete analogue of diffeomorphism invariance.

Lagrange–hex projection Representation of the AOL that organises minimal displacement modes into hexagonally structured layers labelled by distances $\sqrt{n}$.

Ghost layer Structured pattern of allowed lattice moves at a fixed displacement scale $\sqrt{n}$ in the Lagrange–hex projection.

Lagrange–hex ghost kernel Minimal set of displacement classes $\{\sqrt{n} \mid n = 1, \ldots, 6\}$ and their local curvature and adjacency profiles, regarded modulo PAL–diffeomorphisms.

Background independence Requirement that physical observables do not depend on a fixed background geometry or coordinate choice, but only on relational structure. Implemented in PFT by zero–geometry determination.

General relativity (GR) Classical field theory of spacetime curvature described by the Einstein equations. In PFT, GR arises as an infrared projection of PAL–constrained curvature dynamics.

Quantum field theory (QFT) Framework describing particles and interactions as excitations of fields on a spacetime background. In PFT, QFT arises as an infrared projection of PAL–constrained cascades on the AOL.

Appendix B — PFT Internal Bibliography

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Appendix C — Notes on PAL–Diffeomorphism Structure

This appendix collects brief mathematical remarks on the structure of PAL–diffeomorphisms.

C.1 Group–like properties

The set of PAL–diffeomorphisms between AOL completions satisfying fixed structural constraints has group–like properties:

- Composition of two PAL–diffeomorphisms is again a PAL–diffeomorphism.
- The identity map is a PAL–diffeomorphism.
- Each PAL–diffeomorphism has an inverse that is also a PAL–diffeomorphism.

Thus PAL–diffeomorphisms form a group acting on the space of completions.

C.2 Orbits of completions

A completion orbit is the set of all completions reachable from a given completion by PAL–diffeomorphisms. The completion–equivalence theorem implies that all completions in an orbit are physically indistinguishable. The physically relevant configuration space is the quotient of the space of completions by the PAL–diffeomorphism group.

C.3 Relation to continuum diffeomorphisms

In the infrared limit, PAL–diffeomorphisms induce transformations on effective fields and metrics that match continuum diffeomorphisms. The discrete action on cells and curvature assignments becomes a smooth reparameterisation at large scales.

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