The PFT Operator Algebra is Closed: Operator Closure Under Phase Alignment Lock (PAL)

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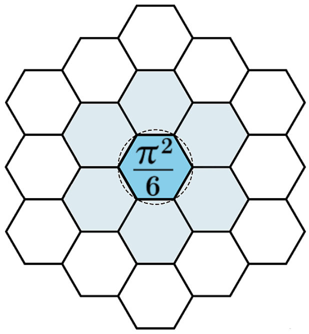

Abstract

Pattern Field Theory (PFT) describes dynamics on the Allen Orbital Lattice (AOL), a prime-indexed curvature lattice. Phase Alignment Lock (PAL) imposes phase-neutrality constraints on prime-indexed lattice faces and stabilises operator interactions.

This paper proves that the five core PFT operators — transport, curvature-weighted derivative, recursion, cross-network coupling, and global evolution — form a closed operator algebra under PAL coherence.

For all operators O_i, O_j in the hierarchy,

$$[O_i, O_j] = \sum_k c_{ij}^k(\theta, p)\, O_k,$$

with structure constants expressed as convergent prime harmonic sums shaped by the AOL spectral bound. This closure holds for all hierarchy versions (v35–v44) and establishes the algebraic backbone underlying gravitational, quantum, cascade, and CCCT (Cross-Coherent Cascade Theory) branches in Pattern Field Theory.

1 Introduction

Pattern Field Theory (PFT) is a field-based lattice framework in which all pattern evolution occurs on the Allen Orbital Lattice (AOL). The AOL is a discrete orbital-curvature lattice with:

- prime-indexed directions,
- curvature weights on edges and faces,
- phase fields that govern coherence,
- hierarchical recursion depth.

Dynamics on the AOL are organised by five operators:

$$\mathcal{O} = \{\mathcal{T}, \mathcal{C}, \mathcal{R}, \mathcal{N}, \mathcal{G}\}.$$

These represent:

- $\mathcal{T}$: transport,
- $\mathcal{C}$: curvature-weighted derivative,
- $\mathcal{R}$: recursion,
- $\mathcal{N}$: cross-network coupling,
- $\mathcal{G}$: global evolution.

Across all PFT versions (v35–v44), these operators retain the same structure, suggesting they are generators of a closed algebra. This paper proves the closure formally.

Phase Alignment Lock (PAL)

Phase Alignment Lock (PAL) is the coherence criterion requiring that divergence of flux across prime-indexed faces is exactly zero. PAL stabilises interactions and ensures operator combinations remain inside the hierarchy.

Allen Orbital Lattice (AOL)

The Allen Orbital Lattice (AOL) is a discrete orbital-curvature lattice seeded by prime geometry. AOL defines:

- curvature weights κ^μ,
- prime-indexed axes,
- PAL phase structure,
- recursion geometry.

The closure theorem depends critically on PAL coherence and the spectral bounds of the AOL.

2 The Five Operators On the AOL

2.1 Transport Operator $\mathcal{T}$

A discrete directional transport with weights $v^\mu(x)$:

$$(\mathcal{T}\phi)(x) = \sum_\mu v^\mu(x)\nabla_\mu\phi(x).$$

2.2 Curvature-Weighted Derivative $\mathcal{C}$

Weighted by local curvature:

$$(\mathcal{C}\phi)(x) = \sum_{\mu} \kappa^{\mu}(x)\nabla_{\mu}\phi(x).$$

2.3 Recursion Operator $\mathcal{R}$

Acts on recursion depth r:

$$(\mathcal{R}\phi)_r = \phi_{r+1} - F_r(\phi_r).$$

2.4 Cross-Network Coupling $\mathcal{N}$

Links networks $m \to n$:

$$(\mathcal{N}_{mn}\phi)_m = K_{mn}[\phi_n].$$

2.5 Global Evolution $\mathcal{G}$

Integrated evolution:

$$\mathcal{G} = \alpha_T\mathcal{T} + \alpha_C\mathcal{C} + \alpha_R\mathcal{R} + \alpha_N\mathcal{N}.$$

3 PAL-Coherence and Divergence Neutrality

Definition 1 (PAL-Coherent Configuration). *A field ϕ is PAL-coherent when divergence of its flux across every prime-indexed face S_p satisfies:*

$$\nabla \cdot \mathcal{F}(\partial S_p) = 0.$$

Lemma 1 (PAL Curvature Neutrality). *On PAL-coherent regions:*

$$\nabla_{\mu}\kappa^{\mu}(x) = 0.$$

This forces cancellation of curvature-induced transport–derivative imbalances and is essential for operator algebra closure.

4 Prime-Harmonic Structure Coefficients

For two operators O_i and O_j, PAL introduces a phase θ_{ij}, and the AOL assigns a prime index p.

Structure constants take the form:

$$c_{ij}^{k}(\theta, p) = \sum_{p\in\mathbb{P}} w_{ij}^{k}(p)\,\frac{\sin(2\pi p\theta_{ij})}{p^{\sigma}}, \qquad \sigma > 1.$$

These converge due to the AOL's prime-indexed spectral bound.

5 Main Closure Theorem

Theorem 1 (Operator Algebra Closure under PAL). *For all $O_i, O_j \in \mathcal{O}$,*

$$[O_i, O_j] = \sum_{k} c_{ij}^{k}(\theta, p)\,O_k,$$

with structure constants given by convergent prime harmonic sums tied to AOL curvature and PAL phase alignment.

Sketch. **1. Transport–Curvature Commutator.**
PAL curvature neutrality removes divergent curvature terms:

$$[\mathcal{T}, \mathcal{C}] = \beta_{TC}(\theta, p)\mathcal{T} + \gamma_{TC}(\theta, p)\mathcal{C}.$$

2. Recursion–Network Commutator.
Boundary terms collapse into the global evolution operator:

$$[\mathcal{R}, \mathcal{N}] = \delta_{RN}(\theta, p)\mathcal{G}.$$

3. Remaining brackets.
All others reduce similarly using PAL neutrality and AOL spectral bounds.

Thus the algebra closes on $\mathcal{O}$. □

6 Prime-Harmonic Commutator Example

$$c^{\mathcal{G}}_{\mathcal{R}\mathcal{N}}(\theta_{mn}) = \sum_{p\in\mathbb{P}} \frac{\sin\left(2\pi p\, \frac{m-n}{m+n}\right)}{p^{\sigma}}.$$

This links recursion depth (m, n) to prime-indexed AOL geometry.

7 Representations

Gravitational Sector

Dominated by $(\mathcal{T}, \mathcal{C})$.

Quantum-Like Sector

Dominated by $(\mathcal{R}, \mathcal{N})$ interactions.

CCCT Sector

Cross-Coherent Cascade Theory (CCCT) emerges from recursion–network–global interplay.

8 Conclusion

The Pattern Field Theory operator hierarchy forms a closed algebra under Phase Alignment Lock on the Allen Orbital Lattice. This result supplies the algebraic backbone for unification across PFT regimes.

Appendix A — Glossary of Terms

PFT — Pattern Field Theory Unified field framework developed on the Allen Orbital Lattice.

AOL — Allen Orbital Lattice Prime-indexed, curvature-aware orbital lattice where all Pattern Field Theory patterns evolve.

PAL — Phase Alignment Lock Coherence condition requiring divergence-neutrality on prime-indexed faces.

CCCT — Cross-Coherent Cascade Theory Pattern Field Theory branch describing cascades and coherence collapse.

Transport Operator $\mathcal{T}$ Directional evolution via discrete lattice transport.

Curvature Operator $\mathcal{C}$ Derivative weighted by Allen Orbital Lattice curvature.

Recursion Operator $\mathcal{R}$ Evolution in recursion depth or cascade index.

Cross-Network Operator $\mathcal{N}$ Couples subnetworks or layers on the Allen Orbital Lattice.

Global Evolution Operator $\mathcal{G}$ Integrated sum of other operators controlling net evolution.

Prime Harmonic Coefficient Series of form $\sum_p \sin(2\pi p\theta)/p^\sigma$.

Hierarchy Versions v35–v44 Internal Pattern Field Theory development sequence refining operator structure.

Appendix B — PFT Internal Bibliography

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Document Timestamp and Provenance

This paper extends the Pattern Field Theory operator framework developed through Pattern Field Theory's dated research chain beginning **May 2025**, including server logs, cryptographic hashes, and versioned texts on `PatternFieldTheory.com`.