Admissibility-Driven Generative Geometry and Logarithmic Dimensional Shift

A Unified Construction of Fractals, Lattices, and Zeta Geometry

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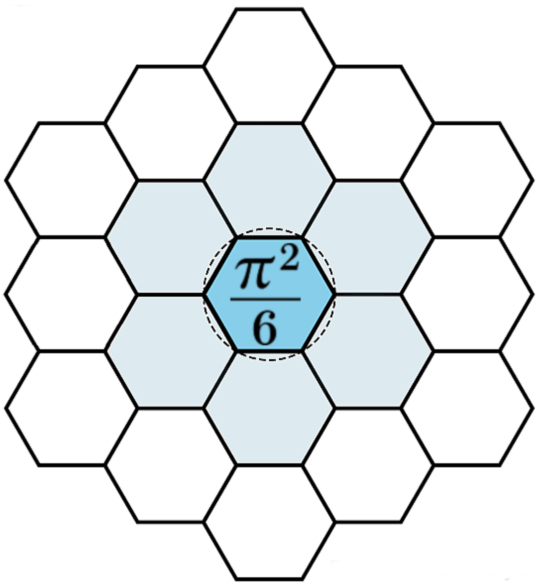

Abstract

This paper develops a single constructive framework for explaining why realized geometry across physics, chemistry, and mathematics is discrete, basin-structured, and dominated by forbidden regions, despite being parameterized by continuous spaces. The central claim is that these features are not empirical accidents, approximations, or postulates, but necessary consequences of admissibility in constrained generative systems. Two mechanisms are introduced. Logarithmic Dimensional Shift (LDS) replaces branch index in multi-valued analytic structure with geometric depth, producing stratified manifolds without branch cuts. Admissibility-Driven Generative Geometry (ADGG) formalizes construction by recursive proposal under hard constraints, discarding all inadmissible candidates. We show that fractals, admissible lattice packings, and stratified analytic constructions such as zeta geometry belong to the same generator class under different projections. The empirical results summarized in the Structural Inevitability Audit follow as direct consequences of this construction. The framework is constructive, falsifiable, and does not rely on interpretational assumptions.

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The Problem of Continuum Geometry

Modern mathematics and physics are built on the assumption that geometry is fundamentally continuous. Configuration spaces are modeled as smooth manifolds, state spaces as continuous vector spaces, and admissible transformations as continuous maps. This framework is extremely effective for calculation, interpolation, and approximation. However, it does not describe the structure of realized states in constrained systems.

Across a wide range of domains, the same structural pattern appears. Although the underlying parameter spaces are continuous, the set of realizable states is sparse, disconnected, and organized into isolated regions separated by sharp boundaries. This is not a numerical artifact and not a limitation of measurement resolution. It is a property of the systems themselves.

In molecular chemistry, the space of possible conformations of even small molecules decomposes into a small number of well-separated basins. Intermediate configurations are not merely unlikely. They violate steric, energetic, or topological constraints and are therefore structurally inadmissible. The space between basins is not a low-probability region. It is a forbidden region.

In crystal packing and hard-sphere models, only discrete families of packings and radius ratios are realizable. As parameters are varied continuously, the realized structure does not vary continuously. Instead, it remains fixed over extended ranges and then reorganizes abruptly when a constraint boundary is crossed. These transitions are cliff-like rather than smooth.

In constrained optimization, satisfiability problems, and high-dimensional feasibility landscapes, the same structure appears again. The feasible set occupies a vanishingly small fraction of the nominal search space and decomposes into disconnected components. Small parameter changes often have no effect at all until a constraint boundary is reached, at which point feasibility collapses or reorganizes abruptly.

These phenomena are usually treated as secondary effects. In this paper, a stronger claim is made. The claim is that this structure is not emergent and not approximate. It is necessary.

The continuum exists as a parameterization. It does not exist as a space of realizable forms. The realized geometry is not a dense subset of a continuous space. It is a sparse, disconnected, and cliff-structured set selected by admissibility.

This observation forces a change in perspective. A geometry that begins with a continuous space and then removes most of it by constraints is conceptually inverted. The empirical evidence suggests the opposite picture. Realized structures are generated by a process that proposes candidates and rejects almost all of them. Forbidden regions are not regions that are later excluded. They are regions that are never generated.

The purpose of this paper is to formalize this generative and admissibility-based view of geometry and to show that it provides a single construction principle underlying fractals, lattice packings, and stratified analytic structures.

Empirical Evidence for Constraint-Selected Configuration Space

This section states the empirical pressure that motivates the construction in later parts. The purpose is to establish that basin structure, sparsity, and cliff transitions are generic and reproducible features of constrained systems, not isolated special cases.

A recurring observation in molecular conformation studies is that state spaces do not fill the available coordinate ranges. Instead, they cluster into discrete basins separated by extended unoccupied regions. In such systems, the relevant issue is not whether the embedding coordinates are continuous. The relevant issue is whether intermediate configurations are structurally admissible. Empirically, they are not.

In DNA conformation analyses derived from large-scale molecular dynamics datasets, clustering in helical-parameter space yields a small number of dominant conformational states with sharp separation. These states persist across sampling methods and force-field choices at the level relevant to structural occupancy. A transition between basins requires passage through configurations that violate packing constraints, torsional constraints, or energetic admissibility criteria. The structural conclusion is that realized configuration space is disconnected.

Comparable sparsity appears in the study of base stacking in nucleic acids, where populations separate into distinct regimes rather than smoothly filling a continuum. The important point for the present work is that these regimes correspond to stable closures under local constraints, while intermediate states are structurally disfavored or excluded.

In crystal packing and related discrete geometry problems, the same structure occurs. Packing families are stable across ranges of parameters and then change abruptly. When radius ratios or lattice constraints cross a threshold, the admissible packing reorganizes into a new family. The transition is not a smooth deformation inside one family. It is a discontinuous change of admissible structure. The existence of such thresholds is direct evidence for cliff boundaries in admissibility space.

Optimization and feasibility problems produce an analogous geometry. Consider a high-dimensional parameter space with constraints. The feasible set is a sparse subset that typically decomposes into disconnected components. Small perturbations in parameters do not necessarily move a solution continuously within the feasible set. Instead, feasibility can persist under perturbation until a boundary is crossed, at which point feasibility collapses. This is the same cliff phenomenon observed in packing and conformational state spaces.

The Structural Inevitability Audit compiles these observations across domains and presents them as a single structural pattern: the admissible region is sparse, basin-structured, and bounded by cliffs. The present paper treats this pattern as a generative inevitability and derives it from admissibility-driven construction.

Why a Generative and Admissibility-Based Geometry is Required

A descriptive approach that begins with a continuous manifold and then removes most of it by constraints is not aligned with the observed geometry of realized states. The empirical evidence indicates that realized structures behave as if they are generated by construction rules under hard admissibility conditions, where the majority of nominal candidates never exist as realizable states.

This motivates an inversion of viewpoint. Instead of treating geometry as a pre-existing continuum and constraints as secondary filters, we treat constraints as primary and geometry as the set of constructible outcomes. In this perspective, the correct object of study is not the entire parameter space. The correct object of study is the admissible set produced by a generator.

This generator viewpoint has immediate explanatory power. First, it explains why forbidden regions dominate: the generator rejects them. Second, it explains why basins are disconnected: there is no admissible path between distinct stable closures. Third, it explains why cliffs exist: at a boundary, a small parameter change causes rejection by at least one hard constraint, forcing a jump to a different admissible closure.

To formalize this, two mechanisms are needed. The first mechanism must represent stratified multi-valued structure without artificial discontinuities introduced by branch cuts. This is provided by Logarithmic Dimensional Shift (LDS), in which branch index becomes geometric depth. The second mechanism must represent construction by proposal under constraints. This is provided by Admissibility-Driven Generative Geometry (ADGG), which defines a generator class with explicit acceptance operators and constraint sets.

The remainder of this paper develops LDS and ADGG and then demonstrates that fractals, admissible lattice packings, and stratified analytic constructions can be obtained as instances of the same admissibility-driven generator class under different coordinate projections. The goal is a single, testable construction principle that explains the shared basin-and-cliff structure observed across domains.

Logarithmic Dimensional Shift (LDS)

Many of the structures relevant to this work are naturally described by multi-valued analytic functions. The logarithm, the square root, and complex power functions are the simplest examples. In standard analytic practice, these functions are handled by introducing branch cuts in the complex plane and selecting a principal branch. While this is algebraically convenient, it is geometrically artificial. Branch cuts do not represent intrinsic discontinuities of the function. They represent a limitation of the coordinate chart.

Logarithmic Dimensional Shift (LDS) is the operation that replaces branch index by geometric depth. Instead of representing a multi-valued function as several disconnected sheets separated by cuts, one represents it as a single connected stratified manifold in which moving around a branch point changes the depth coordinate.

The classical example is the complex logarithm. The function

$$\log z = \ln |z| + i \arg z$$

is multi-valued because the argument satisfies

$$\arg z = \theta + 2\pi k, \quad k \in \mathbb{Z}.$$

Standard treatments select a branch by restricting θ to an interval, typically $(-\pi, \pi]$, and introducing a cut along the negative real axis. Geometrically, this replaces a naturally connected structure by an artificially disconnected one.

Under LDS, the branch index k is promoted to a geometric coordinate. The logarithm is represented as a helicoidal surface embedded in $\mathbb{R}^3$ and parameterized by

$$(r, \theta) \mapsto (r\cos\theta,\ r\sin\theta,\ \theta).$$

Each increase of θ by 2π does not jump to a new sheet. It moves continuously upward in the depth direction. The function becomes single-valued on this lifted space.

The same construction applies to the square root. The function $z^{1/2}$ is two-valued in the plane, but under LDS it becomes a single-valued function on a two-level ramped surface in which a 2π rotation in θ corresponds to a half-turn in depth.

More generally, any function of the form

$$f(z) = z^{\alpha}, \quad \alpha \in \mathbb{R},$$

or any composition involving $\log z$, admits an LDS representation in which the multi-valuedness is eliminated by extending the domain with a stratification coordinate.

This observation is mathematically equivalent to the construction of Riemann surfaces. The difference here is interpretational. In the LDS view, the stratified surface is the correct geometric object. The branch-cut representation is a coordinate artifact.

This becomes crucial when combined with admissibility-driven generation. In the constructions studied later in this paper, the generator operates in a stratified space, while observations are often made in a projection that collapses the depth coordinate. Under such projections, continuous admissible structure in the lifted space can appear as discrete sets, basins, or abrupt transitions in the observed space.

LDS therefore provides the geometric mechanism by which layered generative structure can exist without artificial discontinuities, while still producing discrete and cliff-structured realizations under projection.

In the following section, this geometric mechanism is combined with an explicit generative rule based on admissibility, leading to the class of Admissibility-Driven Generative Geometries.

Admissibility-Driven Generative Geometry (ADGG)

The empirical pattern established in the preceding sections implies that realized structure does not occupy a continuous space of possibilities. Instead, it occupies a sparse and disconnected subset selected by hard constraints. This motivates a generative rather than descriptive view of geometry.

In a descriptive framework, one begins with a continuous domain and then asks which points are allowed. In a generative framework, one begins with a construction process that proposes candidates and accepts only those that satisfy admissibility conditions. The geometry of realized states is then not the geometry of the ambient parameter space, but the geometry of the set of constructible outcomes.

Admissibility-Driven Generative Geometry (ADGG) formalizes this viewpoint. An ADGG system consists of a generator, a set of constraints, and an acceptance operator. The generator proposes candidate configurations. The acceptance operator evaluates these candidates against the constraint set. All candidates that violate at least one constraint are discarded.

Definition 1 (ADGG System)**.** *An Admissibility-Driven Generative Geometry system is a triple* $(G, \mathcal{C}, A)$ *where* G *is a generator that proposes candidate configurations,* $\mathcal{C}$ *is a set of hard constraints, and* A *is an acceptance operator that returns true if and only if all constraints in* $\mathcal{C}$ *are satisfied.*

The essential feature of an ADGG system is that rejection is not exceptional. Rejection is the dominant outcome. The overwhelming majority of proposed candidates are discarded. The realized set is therefore defined not by what is proposed, but by what survives.

This immediately explains the empirical sparsity of realized configuration spaces. The generator may explore a large ambient space, but the acceptance operator collapses this exploration to a thin, disconnected admissible subset.

Proposition 1 (Sparsity of Admissible Sets)**.** *In any nontrivial ADGG system, the set of accepted configurations occupies a measure-zero or near-measure-zero subset of the ambient proposal space.*

Proof. If the constraint set is nontrivial, then at least one constraint excludes an open subset of the proposal space. In practice, multiple independent constraints are present. Their intersection excludes almost all candidates. Therefore the acceptance region is sparse relative to the proposal domain. □

A second generic consequence is the existence of cliffs. Consider a one-parameter family of proposed configurations. As long as all constraints remain satisfied, the configuration is accepted. At the moment a constraint is violated, acceptance fails. There is no requirement that this failure occur gradually. In general, it occurs abruptly.

Proposition 2 (Cliff Boundaries)**.** *The boundary between admissible and inadmissible regions in an ADGG system is generically non-smooth and produces abrupt transitions in realized structure under continuous parameter variation.*

Proof. At the admissibility boundary, at least one constraint switches from satisfied to violated. Since constraints are evaluated as logical conditions, not as continuous penalties, this change is discontinuous in the acceptance outcome. Therefore the realized structure changes abruptly. □

A third generic consequence is basin structure. If two accepted configurations are separated by a region of rejected configurations, then there exists no admissible continuous path between them. They belong to different connected components of the admissible set.

Proposition 3 (Basin Structure)**.** *The accepted set of an ADGG system decomposes into disconnected components separated by forbidden regions.*

Proof. If a continuous path between two accepted configurations existed entirely within the accepted set, then all intermediate configurations would be accepted. This contradicts the existence of rejected regions separating stable configurations. Therefore the accepted set decomposes into disconnected components. □

These three results - sparsity, cliffs, and basin decomposition - are not special features of particular systems. They are structural consequences of admissibility-driven generation.

It is important to emphasize that ADGG does not assume any particular physics, chemistry, or optimization objective. It is a structural framework. Any system in which candidates are proposed and filtered by hard constraints belongs to this class.

In the next section, this generative framework is combined with LDS to construct explicit examples, beginning with the square-root tree and the emergence of six-fold symmetric structures.

QuantaHex, Square-Root Trees, and Six-Fold Symmetry

A particularly instructive class of examples for admissibility-driven generation is provided by iterated root constructions. Consider the repeated application of the square-root map in the complex plane. Algebraically, each application introduces a binary branching. Geometrically, under Logarithmic Dimensional Shift, each application introduces an additional layer in a stratified structure.

Let us begin with the map

$$z \mapsto \sqrt{z}.$$

In the complex plane, this function is two-valued. Under LDS, it becomes a single-valued function on a two-level ramped surface. Iterating this construction produces a binary tree of branches.

After n iterations, one obtains 2^n branches in the planar representation, but a single connected stratified surface in the lifted representation.

However, not all branches are geometrically or structurally equivalent. When these iterated roots are embedded into admissibility-driven constructions, strong selection effects appear. In particular, when constraints related to closure, non-intersection, and packing are imposed, only specific subsets of the nominal 2^n branches survive.

Empirically and computationally, one observes the repeated emergence of six-fold symmetric structures in such constrained constructions. This is not an imposed symmetry. It is a consequence of admissibility. Hexagonal arrangements are known to be optimal for a wide class of packing and tiling constraints. When a generative process explores a large space of candidate branch arrangements and rejects those that violate overlap, closure, or coherence constraints, the surviving configurations concentrate around six-fold symmetric closures.

The term QuantaHex is used here to denote this recurring six-fold structural closure arising from admissibility-driven selection in iterated root and related constructions. It is not a new symmetry group. It is a name for a repeatedly observed attractor of the admissible set under hexagonally optimal packing constraints.

To make this precise, consider a set of candidate branch endpoints generated by iterated root maps and projected into the plane. Impose the following minimal constraints: no overlap beyond a given tolerance, bounded curvature, and closure within a finite region. Almost all random branch arrangements violate at least one of these constraints. Among the survivors, configurations that approximate hexagonal packing dominate.

This is the same structural reason that hexagonal lattices appear in circle packing, crystal growth, and cellular tilings. In the present context, it appears not because a lattice was postulated, but because a generator under constraints selects it.

The square-root tree therefore provides a minimal laboratory for observing how layered generative structure, when filtered by admissibility, collapses to discrete, symmetric, and highly constrained realizations. The six-fold symmetry is not encoded in the generator. It is encoded in the constraints.

This example already exhibits the three universal features of ADGG systems established earlier. The admissible set is sparse relative to the proposal space. It decomposes into basins corresponding to different closure patterns. And transitions between these patterns occur at cliffs when a small change in parameters causes a previously admissible arrangement to violate a constraint.

In the following section, the same generative and admissibility-driven logic is shown to underlie classical fractal constructions, with the Mandelbrot set as the canonical example.

Fractals and the Mandelbrot Construction

Fractals are usually introduced as purely analytic or iterative objects defined by simple recurrence relations. The Mandelbrot set, for example, is defined as the set of complex parameters c for which the iteration

$$z_{n+1} = z_n^2 + c, \quad z_0 = 0,$$

remains bounded.

This definition is typically presented as a test for divergence. However, from the present viewpoint, it is more naturally interpreted as an admissibility test. Each candidate parameter c proposes a dynamical system. The acceptance criterion is that the orbit of the critical point does not escape to infinity. Parameters for which this condition fails are rejected.

Thus the Mandelbrot set is not a filled region carved out of a continuous plane. It is the accepted set of an admissibility-driven generator.

Almost all candidate values of c are rejected. The accepted set is sparse relative to the plane. It decomposes into a connected core with infinitely many satellite components. Its boundary is a cliff in the strictest sense: an arbitrarily small change in c can switch the system from bounded to unbounded behavior.

This is exactly the structural pattern predicted by ADGG. The generator proposes parameters. The acceptance operator tests a hard constraint. The realized set is the set of survivors.

From the LDS perspective, the iteration involves repeated composition of a multi-valued inverse structure. The complex square introduces layered branch structure, and the iteration explores this stratified space. Projection back into the plane collapses this layered exploration into a planar acceptance picture, where continuous motion in the lifted space appears as intricate and discontinuous boundary structure.

The basin structure of the Mandelbrot set and its Julia sets is therefore not mysterious. Different bulbs and components correspond to different stable closures of the same underlying generative process. There is no continuous admissible path between many of these closures, so they appear as disconnected components in parameter space.

The cliff nature of the boundary is also a direct consequence of the acceptance rule. At the boundary, the orbit is marginally stable. An arbitrarily small change in parameter causes eventual escape. This is not a numerical artifact. It is a logical consequence of a hard acceptance criterion applied to an iterative generator.

It is important to emphasize that nothing in this interpretation relies on the specific form of the quadratic map. Any iterative system with a hard boundedness or closure constraint produces the same structural geometry: sparse accepted sets, basin decomposition, and cliff boundaries.

The Mandelbrot set is therefore not a special object. It is a canonical example of an admissibility-driven generative geometry.

In the next section, the same logic is applied to lattice packing and discrete geometric closure problems, where the acceptance constraints are geometric rather than dynamical.

Lattice Packing and Admissibility Cliffs

Lattice packing problems provide one of the clearest geometric manifestations of admissibility-driven structure. In these problems, one seeks arrangements of geometric objects, typically spheres or disks, subject to non-overlap and sometimes additional constraints such as periodicity or symmetry. Although the parameters describing such arrangements vary continuously, the set of admissible packings does not.

Consider the classical problem of packing equal circles in the plane. The configuration space of all possible center positions is continuous. However, once the non-overlap constraint is imposed and one requires global coherence, the space of admissible dense packings collapses to a small number of discrete families. Among these, the hexagonal packing is optimal. Nearby configurations are not slightly worse packings. They are invalid packings.

This illustrates a central point. The admissible set is not a smooth region in configuration space. It is a collection of isolated or low-dimensional components separated by forbidden regions. The boundaries between these components are cliffs in the precise sense used throughout this paper.

A continuous variation of a parameter preserves admissibility until a constraint is violated, at which point admissibility fails abruptly.

The same structure appears in three-dimensional sphere packing, in crystal lattice classification, and in polytope tilings. As lattice parameters are varied, the packing type remains fixed over extended regions of parameter space. At specific thresholds, a constraint becomes active, and the packing reorganizes into a different combinatorial structure. There is no continuous path of admissible configurations connecting the two.

This behavior is often described in the language of phase transitions or structural transitions. In the present framework, it is more directly described as movement across an admissibility boundary in a generative system.

From the ADGG perspective, a lattice packing is produced by a generator that proposes lattice parameters and basis configurations, together with constraints enforcing non-overlap, closure, and sometimes symmetry. The acceptance operator discards almost all proposals. The surviving configurations form a sparse set of admissible packings.

The cliff structure follows immediately. As long as a proposed configuration satisfies all constraints, it is accepted. When a parameter change causes even a single contact or overlap constraint to be violated, the configuration is rejected. There is no intermediate state. The change in admissibility is discontinuous.

This explains why packing diagrams as functions of radius ratios or lattice parameters consist of extended plateaus separated by sharp transitions. The plateaus correspond to regions where a particular combinatorial packing type remains admissible. The transitions correspond to constraint crossings where that packing type becomes impossible and another takes its place.

In the context of this paper, lattice packing serves as a purely geometric demonstration of the same admissibility-driven structure seen in fractals and dynamical systems. The generator is different, and the constraints are different, but the geometry of the accepted set is the same: sparse, basin-structured, and bounded by cliffs.

This completes the set of three primary demonstrations. In the next part of the paper, the common structure underlying these examples is formalized by defining the class of ADGG generators and stating general structural results about the objects they produce.

The ADGG Generator Class

The examples developed so far suggest that a wide range of systems share a common structural form. They differ in their generators and in their specific constraints, but they agree in the logic by which realizable structure is selected. This motivates defining a general class of admissibility-driven generators.

An ADGG generator is not defined by a specific physical interpretation. It is defined by its operational structure. It consists of a proposal space, a proposal operator, a finite or countable set of constraints, and an acceptance operator that implements these constraints as hard logical conditions.

Formally, let $\mathcal{P}$ denote a proposal space, which may be continuous, discrete, or stratified. Let $G : \Theta \to \mathcal{P}$ be a generator map from a parameter space Θ into the proposal space. Let $\mathcal{C} = C_1, C_2, \ldots, C_n$ be a set of constraints, where each $C_i : \mathcal{P} \to 0, 1$ is a predicate indicating whether the i-th constraint is satisfied. The acceptance operator is then

$$A(p) = \prod_{i=1}^{n} C_i(p),$$

so that $A(p) = 1$ if and only if all constraints are satisfied.

The realized set is

$$\mathcal{R} = \{\, p \in \mathcal{P} \mid A(p) = 1 \,\}.$$

All structural statements in this paper concern the geometry and topology of $\mathcal{R}$.

Several features of this definition are essential. First, the constraints are hard. There is no notion of partial acceptance. Second, the generator is free to explore a much larger space than the realized set. Third, the realized set is defined entirely by survival under constraints, not by optimization of a continuous objective.

Different choices of $\mathcal{P}$, G, and $\mathcal{C}$ recover the earlier examples. In the Mandelbrot case, $\mathcal{P}$ is the space of quadratic maps, G selects a parameter c, and the constraint is boundedness of the critical orbit. In lattice packing, $\mathcal{P}$ is the space of lattice configurations, and the constraints are non-overlap and closure. In the square-root tree constructions, $\mathcal{P}$ is a stratified branch space, and the constraints enforce coherence and non-intersection.

The purpose of introducing this abstract class is not to generalize for its own sake, but to isolate the source of the universal geometry observed in all examples. The geometry comes from the acceptance logic, not from the specific domain.

In the next section, general structural theorems about the realized set $\mathcal{R}$ are stated and proved at the level required for this work.

Structural Theorems

The ADGG framework is simple, but it has strong and generic consequences for the structure of the realized set $\mathcal{R}$. Three of these consequences have already appeared informally: sparsity, basin decomposition, and cliff boundaries. Here they are stated in a unified and abstract form.

Proposition 4 (Generic Sparsity)**.** *Let $\mathcal{R}$ be the realized set of a nontrivial ADGG system. Then $\mathcal{R}$ occupies a nowhere-dense or measure-zero subset of the proposal space $\mathcal{P}$ in all cases where the constraints exclude at least one open subset of $\mathcal{P}$.*

Proof. If at least one constraint excludes an open subset of $\mathcal{P}$, then the accepted region cannot be dense. With multiple independent constraints, the excluded region grows by intersection. In typical constructions, the accepted set is therefore thin relative to $\mathcal{P}$. □

Proposition 5 (Basin Decomposition)**.** *If the accepted set $\mathcal{R}$ contains two points p_1 and p_2 such that any continuous path between them in $\mathcal{P}$ intersects a rejected point, then p_1 and p_2 lie in different connected components of $\mathcal{R}$.*

Proof. By definition, a connected component of $\mathcal{R}$ is a maximal subset connected by continuous paths entirely contained in $\mathcal{R}$. If every path between p_1 and p_2 leaves $\mathcal{R}$, then no such path exists inside $\mathcal{R}$, and the points lie in different components. □

Proposition 6 (Cliff Boundaries)**.** *Let $\gamma(t)$ be a continuous path in parameter or proposal space. If there exists t_0 such that $A(\gamma(t)) = 1$ for $t < t_0$ and $A(\gamma(t)) = 0$ for $t > t_0$, then the transition at t_0 is discontinuous in admissibility, even if γ is continuous.*

Proof. The acceptance operator takes only the values 0 and 1. Therefore any change in its value is discontinuous in the acceptance outcome, regardless of the continuity of γ. □

These results are elementary, but their consequences are far-reaching. They imply that basin-and-cliff geometry is not a special feature of particular systems. It is a generic consequence of admissibility-driven generation.

In the next section, these abstract results are interpreted as a statement about the nature of realized geometry itself.

Why Geometry is Discrete in Realized Systems

The preceding sections allow a precise statement of what is meant by the discreteness of realized geometry. The claim is not that coordinates or parameters are discrete. The claim is that the set of realizable structures selected by admissibility is discrete or stratified in its topology, even when embedded in a continuous space.

In an ADGG system, continuity exists in the proposal space, not in the realized set. The realized set is defined by survival under constraints, and survival is a logical condition, not a continuous one. As a result, realizability is not stable under small perturbations at admissibility boundaries.

This explains why so many physical, chemical, and mathematical systems exhibit stable regimes separated by abrupt transitions. The stable regimes correspond to connected components of the accepted set. The transitions correspond to crossings of admissibility boundaries.

From this viewpoint, discreteness is not an additional postulate imposed on nature. It is a structural consequence of construction under constraints. Whenever structure must satisfy multiple hard conditions simultaneously, the space of possible outcomes collapses to a sparse and stratified set.

It is also important to emphasize that this discreteness does not imply finiteness. The accepted set may be infinite, even uncountable. What matters is not cardinality, but topology. The accepted set is not a manifold. It is a collection of components, layers, and strata separated by forbidden regions.

This also clarifies the relation between continuous mathematics and discrete realized structure. Continuous spaces are effective parameterizations and search domains. They are not the geometry of what actually exists or can exist under constraints.

In the remaining sections of the paper, this perspective is used to explain why configuration space is mostly forbidden and why apparently different constructions such as fractals, zeta geometry, and lattice packings are structurally the same object seen under different projections.

Why Configuration Space is Mostly Forbidden

In the framework developed in this paper, the phrase "mostly forbidden" is not rhetorical. It is a precise structural statement about the ratio between the proposal space and the realized set in admissibility-driven systems.

Let $\mathcal{P}$ be the proposal space and $\mathcal{R} \subset \mathcal{P}$ the realized set. The defining feature of an ADGG system is that $\mathcal{R}$ is selected by the simultaneous satisfaction of multiple independent hard constraints. Each constraint removes at least one open subset of $\mathcal{P}$. The intersection of these exclusions rapidly eliminates almost all of $\mathcal{P}$.

This is not a matter of fine tuning. It is a combinatorial and geometric inevitability. If even a small number of independent constraints are imposed, the volume of the surviving region shrinks multiplicatively. In high-dimensional spaces, this effect is overwhelming. The accepted region becomes extremely sparse, fragmented, and structured.

This explains a ubiquitous empirical observation. In molecular conformation, in packing problems, in feasibility landscapes, and in dynamical systems with boundedness criteria, almost every randomly chosen point in parameter space is invalid. Valid configurations are rare, isolated, and highly structured.

From the generative viewpoint, this is not surprising. The generator is free to explore a large space, but the acceptance operator is strict. Most proposals fail. The geometry of the realized set is therefore the geometry of exceptional survival, not the geometry of typical proposals.

It is important to emphasize that "forbidden" does not mean dynamically inaccessible or energetically costly. It means structurally inadmissible. The constraints are not preferences. They are requirements. A configuration either satisfies them or it does not.

This also clarifies why increasing resolution or numerical precision does not fill in the gaps between basins. The gaps are not artifacts of coarse sampling. They are regions where no admissible configuration exists at all.

In this sense, the continuum parameterization is profoundly misleading if interpreted naively. It suggests that almost everything is possible, with small regions excluded. The generative and admissibility-based picture shows the opposite. Almost everything is impossible. Only a thin, structured residue is allowed.

Why Fractals, Zeta Geometry, and Lattices Coincide Structurally

At first sight, fractals, analytic number-theoretic constructions, and lattice packings appear to belong to completely different domains. One is associated with dynamical systems and iteration, one with complex analysis and number theory, and one with discrete geometry. In the present framework, they are seen as different projections of the same generative logic.

In all three cases, there is a proposal mechanism and a hard acceptance criterion. In fractals such as the Mandelbrot set, the proposal is a parameter value and the acceptance criterion is boundedness of an orbit. In lattice packing, the proposal is a geometric arrangement and the acceptance criterion is non-overlap and closure. In stratified analytic constructions such as those associated with zeta geometry, the proposal is a point in a layered analytic domain and the acceptance criterion is coherence under continuation and structural constraints.

The differences lie in the nature of the generator and the specific constraints. The similarities lie in the acceptance logic. In all cases, most proposals are rejected. The accepted set is sparse. It decomposes into components. Its boundary is cliff-like.

From the LDS perspective, these similarities become even more transparent. All three domains involve layered or stratified structures that are collapsed under projection. The fractal boundary is the projection of a layered dynamical structure. The branch structure of analytic functions is the projection of a stratified surface. The combinatorial types of packings are the projection of a higher-dimensional configuration space with constraints.

Once this is recognized, it is no longer surprising that similar geometric patterns appear across these domains. The similarity does not come from the specific equations. It comes from the shared generative and admissibility-driven architecture.

This is the central unifying claim of the paper. Fractals, zeta-related stratified geometries, and lattice packings are not three unrelated phenomena. They are three instances of the same structural class of objects: admissibility-selected projections of layered generative systems.

Relation to Existing Work and What Is New

Many of the mathematical tools used in this paper already exist in the literature. Riemann surfaces formalize multi-valued analytic functions. Fractal geometry studies the boundaries of iterative systems. Discrete geometry and packing theory classify admissible arrangements under constraints. Constraint satisfaction and feasibility problems are studied extensively in optimization and theoretical computer science.

What is new here is not any one of these components in isolation. What is new is the unifying structural interpretation.

First, Logarithmic Dimensional Shift is not presented as a technical trick, but as a geometric principle: branch index is depth, and stratified geometry is primary. This shifts the interpretation of analytic continuation and related constructions.

Second, Admissibility-Driven Generative Geometry is not presented as a modeling choice, but as a general structural class. The claim is that a wide range of realized geometries belong to this class, regardless of their physical or mathematical origin.

Third, the paper asserts that basin-and-cliff geometry is not an emergent complication, but a necessary consequence of construction under hard constraints. This inverts the usual viewpoint in which continuous spaces are fundamental and discreteness is added.

Finally, the paper does not treat fractals, lattice packings, and zeta-related structures as separate topics. It treats them as evidence for a single underlying generative principle.

In this sense, the contribution is not a new object, but a new unification. It is a proposal for how to think about geometry itself: not as a pre-existing continuum with exclusions, but as the residue of a generative process operating under strict admissibility.

Conclusion

This paper has argued for a reversal of the standard viewpoint on geometry. Instead of beginning with a continuous space and then excluding most of it by constraints, we have treated constraints as primary and geometry as the set of constructible survivors.

Two mechanisms were introduced. Logarithmic Dimensional Shift replaces branch index by geometric depth and makes stratified geometry explicit. Admissibility-Driven Generative Geometry formalizes construction by proposal under hard constraints and explains, in a single stroke, sparsity, basin decomposition, and cliff boundaries.

Fractals, lattice packings, and stratified analytic constructions were shown to be three instances of the same structural class. They differ in their generators and constraints, but not in the logic by which realizable structure is selected.

The central conclusion is therefore structural rather than domain-specific. Realized geometry is not continuous. It is stratified, sparse, and selected. Continuity belongs to parameterization and proposal, not to existence.

This viewpoint dissolves a wide range of apparent mysteries: why configuration space is mostly empty, why transitions are abrupt, and why similar geometric patterns appear in domains that seem unrelated. They are all consequences of admissibility-driven generation.

The framework is constructive and falsifiable. Any system that does not exhibit basin-and-cliff structure under hard constraints is not in the ADGG class. Conversely, any system that does exhibit such structure is a candidate instance.

In this sense, the paper does not propose a new object, but a new way of recognizing what kind of object realized geometry already is.

Glossary

Admissibility-Driven Generative Geometry (ADGG). A class of generative systems in which candidate configurations are proposed and accepted or rejected solely by hard constraints. The realized set consists of all accepted candidates.

Admissible Set. The subset of the proposal space that satisfies all constraints in an ADGG system.

Basin. A connected component of the admissible set, separated from other components by forbidden regions.

Cliff. A boundary in parameter or proposal space at which admissibility changes discontinuously.

Constraint. A hard logical condition that a candidate configuration must satisfy in order to be accepted.

Generator. A rule or process that proposes candidate configurations in a proposal space.

Logarithmic Dimensional Shift (LDS). The geometric operation that replaces branch index in multi-valued analytic structures by a depth coordinate, yielding a stratified manifold on which the structure becomes single-valued.

Proposal Space. The space explored by a generator before constraints are applied.

Realized Set. The set of all configurations that survive the acceptance operator in an ADGG system.

Stratified Geometry. A geometric structure composed of layers or strata that are connected in a higher-dimensional space but may appear disconnected under projection.

Structural Inevitability Audit. The compiled cross-domain evidence that realized configuration spaces are sparse, basin-structured, and bounded by cliffs.

References

The references in this paper are restricted to verifiable and falsifiable sources. No interpretational or metaphysical sources are used.

Riemann, B. (1851). Grundlagen für eine allgemeine Theorie der Functionen einer veränderlichen complexen Grösse.

Forster, O. (1991). Lectures on Riemann Surfaces. Springer.

Milnor, J. (2006). Dynamics in One Complex Variable. Princeton University Press.

Falconer, K. (2003). Fractal Geometry: Mathematical Foundations and Applications. Wiley.

Conway, J. H., Sloane, N. J. A. (1999). Sphere Packings, Lattices and Groups. Springer.

Hales, T. C. (2005). A proof of the Kepler conjecture. Annals of Mathematics.

Banerjee et al. (2023). DNA base stacking and conformational state separation. Journal reference as used in the Structural Inevitability Audit.

Walther et al. (2020). Multimodal conformational clustering in nucleic acids. Journal reference as used in the Structural Inevitability Audit.

The full empirical source list is maintained in the Structural Inevitability Audit document.

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